VENID Y VED

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Abstract. In modern elementary mathematics some general methods - in fact, only quite a few - are known. One of these main methods is the method of middle element and another one is the principle of an extreme element. Making simple and clear analogies it might be told that in the area of education middle element principle is rather corresponding to the usual classical education while extreme element principle correspond to various challenges, competitions including Olympiads or other activities of creative education. The interplay of them is strikingly challenging in practically all aspects and is connected to all achievements and also to all tensions and collisions including various contradictions and even possible crises in every area of real education.

Keywords: math contests, advanced and standard education, emotions, elements of poetry, attractiveness of details.

Introduction

These lines are based on some fundamental thoughts and ideas, which are strikingly actual and at the same time extremely understandable and provocatively challenging to every real teacher independent of the type of school, country or continent presented – in every place all over the world. The author feels the temptation to say that exactly nowadays the tensions and connections between the ordinary and extraordinary education are reaching such a level, which has never been reached before. It reflects also an eternal span between what we need and what we have achieved. These tensions and connections also do not depend on the level or profile of school but can be influenced by the form of teaching just as we already hear that approach and perspective of the usual primary school in India may look more promising than all well calculated and brilliantly formulated standards of well situated gymnasium located anywhere, say, in the welfare Western society. It's a tautology that you can't teach math in exactly the same way as you teach poetry, but on the other way there are very much in common - much more than one could suggest, especially if you are really applying additional powers and at least some possible resources of education to teaching. Of course you can't apply all or most of them at a time! From that point of view you can even have a feeling that all these arts including math are of a rather similar nature. If they are not of exactly the same nature then they are of a strikingly similar nature or at least they seem to rise from rather neighboring sources. In a classical sense mathematics is strict and exact, this is a seldom place and area where you know exactly what is truth and what is not. This is a seldom place where you have rather clear feelings, which way you are to follow. In math you have the criteria of truth and using these criteria you can filtrate almost everything practically everywhere. But when you start teaching more than 10 persons at a time then all kinds of human factors are influencing you and, afterwards, are assisting as a result all the time long. You can no longer repeatedly say that one must learn math because it is classical or challenging but you must permanently use all your powers in order to make it more understandable or try to employ some technical resources to dispose in order to be able demonstrate some verifiable success. Then you need everything you are able to attach or to involve. It is relative to that feeling when an experienced teacher standing before his class sees that his methods seem to fail at that moment and desperately asks everybody involved and interested: Quo vadis? These lines are also based on long years of teaching activities with usual and gifted high school students and also all life long activities teaching University students and dealing with teachers and other adults, teaching mathematics and not only mathematics. I hope that this will also by some degree reflect everlasting thoughts devoted to the main thing: what does it mean to be at least a bit wise and what does it mean to be really educated? This is a part of eternal question, which name is "What is human education and what the real values of the whole are?" The author is completely aware that he is touching questions that serious countries with their noble education institutions were solving for centuries. For some possible reproaches the author has an explanation in English style saying, "The cat may look at a king". Keeping in mind the tremendous tasks of our Congress and variety of themes considered this might be even modified in form "You and me we all must take a long look". Or otherwise again

Venid y ved.

Usual classical aspects and contents of mathematical events

There are some clear aspects of mathematical events. These events are challenging, leading and guiding all involved girls and boys permanently to new level of understanding of problems exposed and environment involved. First of all, they are increasing the personal trait, which you note at once when you are dealing with a person, but which is difficult to measure in a quantitative way – I mean the deepness of mind. In rather vague words, it is connected with what you can state at once and how deep you are able to feel at once and how quickly you are able to move along afterwards. It could be compared with what kind of car you are driving. Some persons are given extremely powerful cars. They start and move with such a speed, which is so difficult to follow even for the most experienced teacher.

By the way the really good teacher and educator probably (in math especially) starts by internal acceptation of this – I mean the clear understanding that my pupil might be sometimes or often seeing more than me. This is, of course, connected more with psychology of the whole process, which never runs simply.

Other thing, as it is many times mentioned – e.g. by Prof. Agnis Andžans, is the growing role of discrete mathematics when compared with continuous mathematics. That fact you notice even more in various competitions. This is good and not only good. Continuous matters are rather deep – on the other hand they arise from the discrete things. Perhaps the main advantage of the discrete component is the possibility for a simple bright student to enter and almost immediately achieve something – it's so inspiring!

The last thing remains also to inspire educated minds to devote themselves to the art of mathematics. But on other hand, the deepening of mathematically engaged environment is, democratically speaking, even more important. Olympiads are very good for wakening of desire to study mathematics or other exact sciences but strict thinking is so important to a real rising of the general culture of thinking and understanding of the whole.

You cannot replace the exact thinking by words that exact thinking is nice – this is true, this must be said but this alone is not enough!

Not so usual aspects and contents of mathematical events – form and adoption

Not so usual and widely discussed aspects of mathematical events and competitions are also rather well known. Naturally they are known not to such remarkable extent and degree as the classical advantages of the developing of basic mathematical ideas and skills. Classical advantages of mathematical contest are those leading to the deepening and perfection of content of basic fundamental ideas of modern elementary mathematics. Not so usual aspects are related to the feeling of form and to perfection of it in global and also in details. We intend to speak about psychological matters later. Now we would like to notice that this perfection in details is the obvious place where ars mathematica gives the hand to ars poetica. These things are nowadays seldom discussed rather. Only the funny remark about mathematicians having more fantasy than even poets – I really do not understand, why? – is being repeated and cited almost as often as that mathematics is the queen of all sciences.

Psychological perspectives the struggle for interest of listeners would only increase

There are two things – the psychological environment in the competition itself and the struggle for psychological advantages in order to make the problems interesting from the psychological point of view. The author is well enough familiar with the both areas of that huge sample of tensions. I was a deputy leader in more than 10 IMO's starting from Mumbai India 1996 and I've seen to what tension the participants undergo. For years the leading teams delegate additional professionals in order also to smoothen the possible tensions of psychological nature, which always do take place. Sometimes you are standing before the shy looking problem that some years ago were modestly called questions – for example, you may take Problem 3 from IMO in Vietnam 2007. This problem looks similar to the average text problem or to standard Olympiad problem, but was solved by 2 (in words: two) participants. Afterward you can much better understand the feelings of future King David immediately before his struggle with Goliath. Below we present the text of that problem.

Problem 3 (48th IMO Vietnam 2007):

In a mathematical competition some competitors are friends; friendship is always mutual. Call a group of competitors a *clique* if each two of them are friends. The number of members in a clique is called its *size*.
It is known that the largest size of cliques is even. Prove that the competitors can be arranged in two rooms such that the largest size of cliques in one room is the same as the largest size of cliques in the other room.

But let us return from Heaven to Earth and let us also say some words about competition problems there. The psychological problems will never be eliminated so the modern experience teaches us to raise them frankly and openly and discuss them. Such discussions may do no harm and, on the other side, they may be of the great value and importance.

Not in vain in USAMO books and related literature on the introductory pages you will always find clear advice for the possible solver to take his time. It is told never to forget that these problems have seen many hands and minds, each mind adding something in order to make them even more and more perfect than they were. It is told in nice and true words – remember, that you are dealing with perfect products of (collective) perfect human mind – so don't be astonished if at the very beginning you can just stand and stare. So the situation on the initial stage is similar just as in the Don Quixote's struggle against the windmills. Or it is similar to Gulliver's state when he was laying in the Lilliputian land being bounded to the Lilliputian earth till the last lock of hair and when he woke up he was trying to move his small finger – and couldn't.

In general the classical side of contest or any other real mathematical event evidently has very much in common with any other human competition. The difference is that when dealing with the mathematical problems you in fact have possibility to see whether the solution suggested is right or not right – perhaps still (much) as on any other area.

One of the main eternal psychological problems is perception or understanding that something is already achieved. In order to say that in two words let us cite the immortal lines of the verse of Child Innocence.

The Grizzly bear is huge and wild,
It has devoured the infant child,
The infant child is not aware,
He has been eaten by the bear.

Those thoughts lead also to the smallest atomic elements of importance.

Atomic elements of improving math events in details – including poetry

Trying to compress some precious mathematical advice or ideas or comments to the smallest possible size – in formulating, explaining or developing thoughts and ideas – which are of extreme importance also in math competitions you can luckily employ some poetical pattern taking advantage of its perfectly short formulations which make minimal poetical forms relative to the mathematical formulas. After such comparison is more understandable why we still discuss the beauty of mathematical formulas – they are nice because they are laconic. Let us cite the immortal lines of Harry Graham:

"There's been an accident!" they said,
"Your servant's cut in half; he's dead."
"Indeed!" said Mr. Jones, "and please
Give me the half that's got my keys."

Now try to describe using less words the procedure when you are looking for the element which you need, dividing the sample of candidates in two parts afterwards eliminating one of them and regarding another.

Or find clearer words to explain the possible difference between the possible and necessary condition as it might be done using poetical words:

Sir, I'll admit your general rule /That every poet is a fool / But you yourself may serve to show it / That every fool is not a poet. -

Computers and what they do – is that all?

It is difficult to discuss the role of computers because of this being done with increasing intensiveness – starting from the remark that computers are only the sample of metal, but that sample of metal can win against the World Chess Champion, and finishing with the idea that one day they will overtake the ruling of our World. The common truth is that the truth lies in the middle.

It is commonly accepted that the computer when normally applied prolongs the human hands and that you should never forget the possibility of applying the computer in order to make a dull problem from a once complicated exercise of math competition of the highest level.

There are always two sides opposite to each other. There is not only the temptation to apply the computer but also the temptation to avoid it.

Figuratively speaking you always have the possibility to switch on the computer as well as switch it off.

Both possibilities are challenging. Let us give an example when it might be interesting to switch the computer off. This problem is taken from the Saint-Petersburg City Olympiad – these are of highest educational taste and importance and could be recommended to see and enjoy.

Which numbers comprise the greater sample?

(A) 5-digit odd numbers with sum of digits 38;
(B) 5-digit even numbers with sum of digits 36.

The idea of the solution is to construct the matchmaking or prosaically speaking mapping between these 2 samples to see what might happen.

Some personal remarks of possibly extremely subjective nature

At this point the author would like to add that 10 years he is writing a series of articles about simple but challenging problems on some Lithuanian computer magazine Kompiuterija so that he has some feelings about what the computer oriented young minds might think, feel and what they might expect from simple but challenging tasks.

That permanent writing together with all life teaching as well as providing, organizing and selecting and adopting problems for various kind of contests made me to be the happy author of the book “What to do, when you do not know what to do” which originally appeared in Lithuanian in 2006.

Later on. mainly and essentially due to encouraging and support of Professor Agnis Andžans the author translated it into English. The English translation was published in two parts in Riga in 2006 and 2007 in LAIMA series.

One remark about the beauty of exposing problems

Firstly, let us present this problem. It is so psychologically nice in being presented that you at first do not feel any desire to solve it – only read and enjoy – nothing more.
By the way the problem has a unique solution.

A young man walks into a “7-eleven” store and asks for four items. The shop assistant tells him that his bill is 7.11 euros, since the product of four prices is exactly 7.11. The young man explains indignantly that one is supposed to add together the four prices, not to multiply them. “Oh dear!” exclaims the shop assistant and sums the four numbers. But, can you imagine, the right sum turns out to be 7.11 too. How much did each item cost?

It is amazing to see what can be done from a rather simple question – also of some interest – find 4 prices so that their sum is the same as their product.

As far as the author was able to establish, the problem itself is of Polish origin and the adoption, which we have a pleasure to cite is due to V.Ufnarovski, the author of well known book “The mathematical aquarium”.

Several other adoptions due to the author

The author would dare to present some adoption of problems, which are not invented by him. They were selected by the author for two Lithuanian Olympiads for youngsters. These contests were organized as an appendix to the Lithuanian team contest, which has been provided since 1986. In the years when Lithuania and other Baltic republics regained their Independence the Lithuanian Team contest in mathematics inspired the “Baltic Way” team-contest, where all countries of Baltic region together with Iceland participate.

The first set of 5 problems or problems 1-5 are meant for grades 5 and 6 (12-13 years old), and the second five (Problems 6-10) are meant for grades 7 and 8 (13-14 years old) .

1. On the 29th of September 2007 Winnie-the-Pooh attended an open Baron Munchhausen Olympiad in Duntes in which he was expected to solve 20 problems. For each correct solution he was awarded 8 points, whereas 5 points were deducted for an incorrect answer. No points were given or subtracted for the problems Winnie

left unsolved. In the end, Winnie-the-Pooh received 13 points. Can you ever deduce how many problems he solved?

2. Baron Munchhausen regards a 2-digit integer to be *of exceptional importance* in the case when that integer is obtained by adding to the product of its digits the double sum of them and only them. For example, Baron doesn‘t regard the number 49 to be *of exceptional importance* because

$$49 \neq 4 \cdot 9 + (4 + 9) + (4 + 9) = 36 + 13 + 13 = 49 + 13 = 62.$$

(i) Indicate at least one 2-digit integer *of exceptional importance;*
(ii) Find some two 2-digit integers *of exceptional importance*;
(iii) Find all such 2-digit integers *of exceptional importance*.

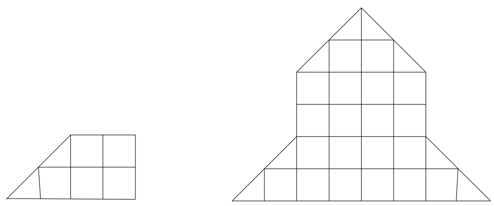
3. Baron Munchhausen accepts a square 3×3 to be *magic*, if the sum in the numbers in each row, column and in each of the main 2 diagonals is the same. Winnie-the-Pooh knows that the square is *magic* but he is not so sure how he could indicate an exact value of x . Help Winnie-the-Pooh to indicate x.

		3
x	4	5

4. The only son of Baron Munchhausen, Stanley, has 2 lorries of different *capacity*. Using the first lorry 3 and the second lorry 4 times Stanley is able to transport less than 30 tons of goods, and using the first lorry 5 and the second one 9 times Stanley is able to transport more than 60 tons of goods.

Which of Stanley‘s lorries is of greater *capacity* and why?

5. Baron Munchhausen claims that it is quite easy to divide each of two figures shown in the picture into 4 equal parts. Is Baron right?



6. (A) Baron Munchhausen deeply believes that it is possible to indicate 4 distinct 4-digit positive integers consisting only of digits 1, 2 and 3 such that any two of these numbers have equal digits in at most one position. Is it really so? Could you ever indicate for him 4 such positive integers.

(B) Baron Munchhausen never thinks that it is possible to indicate 6 such distinct 4 digit positive integers consisting only of digits 1, 2 and 3 such that any two of these numbers have equal digits in at most one position. Is it really so? Could you ever indicate 6 such numbers.

(C) Find the maximum number of distinct 4-digit positive integers consisting only of digits 1, 2 and 3 such that any two of these numbers have equal digits in at most one position.

7 (A) Baron Munchhausen claims that it is impossible to arrange all integers 1 to 16 on a straight line so that the sum of any two adjacent numbers is the square of an integer. Is it indeed so?

(B) Baron Munchhausen claims that it is easily possible to arrange all integers 1 to 16 on a circle so that the sum of any two adjacent numbers is the square of an integer. Is it indeed so?

8 Points K and L are taken by Winnie-the-Pooh on the sides BC and CD of a square $ABCD$ so that $\angle AKB = \angle AKL$. Help Winnie to indicate the true magnitude of $\angle KAL$.

9. (A) Mr Sherlock Holmes together with Dr Watson wish to find all such pairs (x, y) of positive integers x and y such that

$$x^2 - y^2 - x + y = 10.$$

How many and what pairs will they find?

(B) Help them in their attempts if only possible to indicate a pair (x, y) of positive integers x and y such that

$$x^2 - y^2 - x + y = 2007.$$

10. A square consists of 7×7 identical quadratic squares. Some of them Winnie-the-Pooh had coloured black in such a way that number of black squares in each row and in each column is even (possibly 0).

(A)Is it possible for Winnie to colour exactly 4 quadratic squares in such a way that the given condition is satisfied?

(B) Is it possible for Winnie to colour exactly 6 quadratic squares in such a way that the given condition is satisfied?

(C) What number of quadratic squares would be possible for Winnie to colour in that way? Indicate all possible cases.

Some probably optimistic final remarks as conclusions

The author would not intend to speak about the reduction of math lessons in standard school or even not about the decreasing interest in mathematics or other fundamental sciences. The author wouldn't speak about how math is useful. It's the common truth and is repeated, thank God, everywhere. Let us remain concrete and speak about practical matters.

Mathematical Olympiads are useful also because they teach us to do real things and to understand real beauty. They also teach without long words that real things are not easy to realize. At the same time they are witnessing that beauty is possible also in every-day life and elegance may be presented and seen even in the smallest details. You only must come and then you will see.

The author for years had rather a lot to do with organizing and selecting and adopting of problems for various contests and undoubtedly noticed that this whole process is extremely important and also really attractive for young minds – real things like real beauty is attractive and even captivating. The author sometimes thinks that in the future some net of real alternate education may exist also in the form of some contest activity just as it exists now as an additional form of education.

Still in order to raise the interest of students the teacher is expected to use the whole spectrum of everything that is nice, popular and attractive – elements of poetry, adoption of problems and all other possible real and magic elements.

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