

CONCEALING CHOICES TO TEACHERS

Allan Tarp
the MATHeCADEMY.net

Teaching or preaching - this dilemma goes back to the ancient Greek controversy between the sophists advocating enlightenment, and the philosophers advocating patronization. Also today two kinds of schools exist, the North American enlightenment schools educating the people, and EU Humboldt Bildung Counter-enlightenment schools educating the elite for offices. Should teachers be told if they are trained for enlightenment or patronization? Or are they just expected unreflectively to follow the orders of the institution paying their wages?

TWO DIFFERENT KNOWLEDGE INSTITUTIONS

After only one day, an ethnographer will see a fundamental difference between universities in North America and in the EU. At the first place the students begin at age 18, at the second at age 22. At the first place the students have chosen their own combination of modules accessible to all; at the second place they are forced to follow one of several pre-designed educations only accessible to those with the highest marks. At the first place the students already took some university modules at the last year in high school; at the second place this is not possible. At the first place high school is attended by all, and a high percentage goes on to university where around 50% gets a bachelor degree; at the second place only the best half of a year group is allowed to enter high school, and only the best half is allowed to go on to university where only the best half graduates after being forced to finish with a university directed master degree. At the first place some students are supplementing their bachelor with new modules in order to change career e.g. from teaching to engineering; at the second place they have to start all over. Wilhelm von Humboldt seems to hold the key to this difference:

Our universities have a monastic origin, and they have specialized in being centers of higher learning, functions originally given by the Church to monasteries. (..) The form of the university most familiar to us today is mainly a Prussian invention whose architect and champion was Wilhelm von Humboldt. (..) The collegial system and its related peer review structures centered on an effort to gain intellectual freedom from the constraints of theological doctrine and political manipulation. Although addressing this problem was obviously important, the solution adopted has subsequently done much to weaken the social articulation of the university to all groups other than powerful elites. (..) This situation is predictable because the autopoietic research process provides important supports for intellectual freedom but simultaneously opens the door to useless research and academic careerism divorced from attention to important public social issues. (..) Central to our argument is the claim that action research creates the valid knowledge, theoretical development, and social improvements that the conventional social sciences have promised. (Greenwood & Levin in Denzin & Lincoln 2000: 85-89).

So today two different forms of knowledge institutions exist. A Humboldt institution for the elite practicing autopoietic self-reference producing knowledge of little relevance; and an enlightenment institution open to all and providing the public with relevant knowledge.

A HISTORICAL BACKGROUND

To get a better understanding of the two different knowledge institutions we must go back in history. Apart from the gather/hunter economy, the world has witnessed three different economies, a silk&silver economy, a cotton&iron economy and today's knowledge-economy.

The Greek silver mines enabling trade with Far-East luxury goods as silk and spice lasted one hundred years, and financed the Greek democracy housing a controversy between two kinds of knowledge-men, the sophists and the philosophers. The sophists warned that to protect democracy, people should be enlightened to tell choice from nature in order to prevent patronization presenting its choices as nature. The philosophers seeing everything physical as examples of meta-physical forms only visible to them saw patronization as the natural order when conducted by the philosophers educated at Plato's academy. (Russell 1945).

Silver mines in Spain enabled the Romans to finance their empire that collapsed when the Arabs conquered the Spanish silver mines. After the dark Middle Ages, the silk&silver economy was reopened by German silver financing the Italian Renaissance, and by silver found in America. Robbing the slow Spanish silver ships was no problem to the English; finding a route to India on open sea was. Until Newton found that when the moon falls to the earth as does the apple, it is not obeying the unpredictable will of a meta-physical patronizer only attainable through faith, praying and church attendance; instead it is following its own predictable physical will attainable through knowledge, calculations and school attendance.

Bringing Indian cotton to North America created a cotton&iron economy, as well as the Enlightenment period: when an apple obeys its own will people could do the same and replace patronization with democracy. Two democracies were installed, one in US, and one in France. US still has its first republic, France now has its fifth. The German autocracy tried to stop the French democracy by sending in an army. However, the German army of mercenaries was no match to the French army of conscripts only too aware of the feudal alternative to stopping the German army. So the French stopped the Germans and later occupied Germany. Unable to use the army, the German autocracy instead used the school to stop enlightenment spreading from France. Humboldt was asked to create an elite school; and using Bildung as counter-enlightenment he created a school-system leading to the Humboldt University, today using Luhmann System Theory to defend its chosen self-reference as nature.

Inside the EU the sophist warning is kept alive in the French postmodern thinking of Derrida, Lyotard and Foucault warning against patronizing pastoral categories, discourses and institutions presenting their choices as nature. Derrida recommends that pastoral categories be 'deconstructed'. Lyotard recommends the use of postmodern 'paralogy' research to invent alternatives to pastoral discourses. And Foucault uses the term 'pastoral power' to warn against institutions legitimizing their patronization with reference to categories and discourses basing their correctness upon choices claimed to be nature (Tarp 2004).

Sophist and postmodern thinking form the base of 'anti-pastoral sophist research' searching for alternatives to pastoral choices concealed as nature. Looking at reproduction rates in North America and the EU, the US meets the 2.1 child/family that ensures stability; whereas the EU faces 1.5 child/family, in 200 years reducing EU population to 500 mio. * $(1.5/2)^8$ times, i.e. to 50 mio. or 10% . So created by the autocracy to identify the elite for state offices, the Humboldt system well suited a silver&silk economy only needing few to become university students. However, in a knowledge economy needing the majority of each year group to graduate, the Humboldt Bildung system is a disaster wiping out most of the EU-population in 200 years if not changed from a pastoral to an enlightenment system.

ENLIGHTENMENT MATHEMATICS AND PASTORAL MATHEMATICS

In primary school, an enlightenment curriculum focuses on the nature of numbers, operations and calculations, first ‘iconizing’ five sticks in a 5-icon etc., then counting by bundling & stacking reported by cup-writing and decimals and predicted by the ‘recount-formula’ $8 = (8/5)*5 = 1*5 + 3*1 = 1)3) = 1.3*5 = 1.3$ 5s, in accordance with the Piaget ‘from hand to head’ principle of natural learning. Introducing 1digit mathematics allows postponing the ‘cognitive bomb’ 10 until several examples have shown that the icon of the bundle-size is never used since a full bundle will be counted as 1.0 bundles or 10 (Zybartas et al 2005).

The pastoral curriculum introduces the ‘natural numbers’ one by one using the follower-principle. This leads directly to 2digit numbers and place-values by introducing 10 as the follower of 9. Addition is introduced first to revise earlier numbers adding up to the actual number. Then subtraction is introduced as taking away and counting up to. Multiplication and tables follow; and finally division creating a new type of numbers, fractions. (NCTM 2000).

In middle school, an enlightenment curriculum focuses on the nature of per-numbers and triangles. Per-numbers occur when double-counting a quantity in two different units leads to fractions and percentages: 2\$ per 3m = $2\$/3m = 2/3$ \$/m. The recount-formula then enables changing units: $16\$ = (16/2)*2\$ = (16/2)*3m = 24m$. When adding fractions and percentages, the units are included as in integration. Formulas with two unknowns are graphed. With only one unknown, a formula becomes an equation solved by using reversed calculations, first reducing a multiple calculation to a single by placing the hidden parentheses, and then moving numbers to the other side reversing their calculation signs. Geometry is introduced via earth-splitting leading to the right-angled triangle seen as a rectangle halved by a diagonal, where the height and length can be recounted in diagonals as the percentages sin and cos.

The pastoral curriculum enlarges the number domain with fractions, and with decimals and percentages as examples of fractions. Again the order of operations is maintained starting with addition of fractions including factorization of ‘natural’ numbers in prime numbers. Equations are statements about equivalent number-names to be transformed by identical operations aiming at neutralizing numbers. Recounting is called proportionality. Algebraic expressions are introduced to be factorized and simplified, and to be added as algebraic letter fractions. In geometry the focus is on 2- and 3-dimensional forms and translation groups.

In high school, an enlightenment curriculum focuses on adding per-numbers, where adding constant per-numbers leads to power, and adding variable per-numbers leads to integration where middle school’s adding fractions with units, $1/2$ of 2 + $2/3$ of 3 = $3/5$ of 5, together with primary school’s adding stacks in combined bundles, 2 3s + 4 5s = 3.2 8s, is generalized to the area under the per-number m/s graph, $A = \int m/s * s$. Reversed calculation then leads to roots and logarithms, if $x^3 = 10$ then $x = \sqrt[3]{10}$, if $3^x = 8$ then $x = \log_3(8)$; and to differentiation: $2s$ at $3m/s + 5s$ at $?m/s = 7m$ at $4m/s$, $? = (7*4 - 2*3)/5 = \Delta m/\Delta s$.

The pastoral curriculum enlarges the number domain with irrational and real numbers, and the number of operations is enlarged with power, root and log. The function concept is claimed to be the foundation of high school mathematics, and is defined as an example of a relation between two sets. Linear and exponential change is presented as examples of functions. The quadratic function is given an extended treatment. Its graph is studied using translations, and its formula is thoroughly factorized. Calculus is introduced as an example of the limit concept used to define continuity, and the gradient by the first principle, and the integral as a Riemann sum. Geometry introduces coordinate geometry and vector geometry presenting vectors as equivalence sets of parallel arrows with the same length.

CONCEALED CHOICES IN MATHEMATICS EDUCATION

Through the glasses of anti-pastoral sophist research many concealed choices are revealed, rising the question ‘why conceal to teachers these choices between two opposite alternatives?’

Why conceal that ‘education’ is a choice? Education can mean enlightenment aiming at enlightening the outside world to enable students to practice democracy by being able to tell choice from nature; and testing using the real world things and actions it wants to enlighten. Or it can mean Bildung aiming at patronizing students by telling them how the physical world must be understood as examples of metaphysical forms only visible to university graduates who’s patronization therefore should be accepted with servility; and testing using concepts from a pastoral discourse called Didactics claiming to describe the content of Bildung.

Why conceal that ‘school’ is a choice? A school might be a democratic enlightenment-school aiming at enlightening as many as possible as much as possible. Or a pastoral Bildung-school wanting to identify the elite to be educated for offices.

Why conceal that ‘student’ is a choice? It can be a child trying as other mammal children to learn about the outside world through Piagetian adaptation. Or it can be a sexual mature adolescence creating a self-identity as a biographical narrative expanding through gossip, i.e. through statements with known subjects enlightened by the school.

Why conceal that ‘learning’ is a choice? It can be a bottom-up grounded pyramid growing through Piagetian induction and construction. Or it can be top-down metaphysical pyramid trying to be reached by Vygotskian deduction and scaffolding.

Why conceal that ‘teaching’ is a choice? It can be guidance arranging enlightening meetings with categories and facts grounded in the outside world. Or it can be preaching a pastoral discourse claiming to save humans from ignorance.

Why conceal that ‘teacher education’ is a choice? It can consist of a combination of coordinated modules that can be supplemented to facilitate a change in career, taken at a multi-faculty university with instructors carrying a PhD. Or it can be specially designed for a teaching job and impossible to integrate in a different bachelor degree in case of a change in career, taken at a mono-faculty academy where the instructors do not have PhDs.

Why conceal that ‘numerals’ is a choice? They can be icons showing the degree of multiplicity they represent: 5 strokes in the 5-icon etc. Or they can be arbitrary symbols.

Why conceal that ‘10’ is a choice? It can be a short version of writing 1.0 bundle, thus seven = 10 when counting in 7-bundles. Or it can be a pastoral choice ten = 10.

Why conceal that ‘natural numbers’ is a choice? Thus the follower of nine is 10 only when counting in ten-bundles. With seven-bundles we count 5, 6, 10, 11, 12, 13, So counting in seven-bundles, 10 is the follower of six, and the follower of nine is 13.

Why conceal that ‘addition’ is a choice? It can respect the laboratory observation that two stacks can be added side-by-side so that 3 5s and 2 1s become a double-stack of 3.2 5s; or be added on-top so that 2 5s and 1 6s becomes 3.1 5s or 2.4 6s. Or it can be pastoral addition where +5 means finding the fifth follower.

Why conceal that ‘multiplication’ is a choice? It can respect the laboratory observation that $6 \cdot 7$ means 6 7s. Or it can be pastoral multiplication where $6 \cdot 7$ means recounting 6 7s into tens as 4.2 tens, written in a sloppy way as 42 leaving out both the decimal and the unit.

Why conceal that ‘division’ is a choice? It can respect the laboratory observation that $8/2$ means splitting 8 in 2s. Or be pastoral division where $8/2$ means splitting 8 in 2.

Why conceal that ‘subtraction’ is the only operation with only one meaning saying that $8-2$ means ‘from 8 take away 2’? And why conceal that subtraction is the most fundamental

operation? Why conceal that ‘/’ and ‘-’ are not just symbols, but icons, where ‘-4’ shows the dragging away of 4; and ‘/4’ shows the shoveling away of 4s?

Why conceal that ‘equation’ is a choice? It can be reversed calculations, reversing the calculation-signs when moved across the = sign. Or equivalence-statements relating number-names, to be transformed by identical operations aiming at neutralizing the known neighbors.

Why conceal that ‘calculus’ is a choice. It can be adding stacks side-by-side, and per-numbers or fractions carrying units. Or it can be introduced as an example of a limit process.

Why conceal that ‘definition’ is a choice? It can be grounded, defining its concept as an abstraction from examples. Or it can be metaphysical, defining its concept as an example from an abstraction, thus turning grounded mathematics upside down to ‘metamatics’.

Why conceal that ‘proof’ is a choice? It can be a laboratory-proof testing a prediction deduced from a hypothesis. Or it can be a library-proof showing how a statement can be deduced from others statements. Thus ‘ $2+3=5$ ’ is ‘mathematism’ only true in a library, but having countless counter-examples in the laboratory: $2\text{weeks}+3\text{days} = 17\text{days}$, $2\text{m}+3\text{cm} = 203\text{cm}$, etc. Whereas ‘ $2*3=6$ ’ is grounded mathematics true both in the library and in the laboratory by just stating the physical fact that 2 3s can always be recounted as 6 1s.

Why conceal that ‘function’ is a choice? It can follow Euler proposing the name function for calculations containing numbers and variables, becoming interesting after calculus enabled the calculation of changes in, not numbers, but calculations with variable numbers. Or it can present itself as metamatics as an example of a many-to-one set-relation.

Why conceal that ‘algebra’ is a choice? Respecting the Arabic meaning ‘re-uniting’, it can be grounded in the two opposite questions ‘how to unite parts into a total?’ and ‘how to split a total into parts?’ leading to the four different ways of adding and splitting, where + and * add variable and constant unit-numbers, and \int and \wedge adds variable and constant per-numbers; and where a total is split by - and / into variable and constant unit-numbers, and by d/dx and $\sqrt{\log}$ into variable and constant per-numbers. Or be pastoral algebra from above where numbers, operations and equations all are examples of the metaphysical concept ‘set’.

Why conceal that ‘geometry’ is a choice? Respecting the Greek meaning ‘earth-measuring’, it can be grounded geometry from below enlightening the problem of measuring a piece of earth in triangles. Or be pastoral geometry from above showing how physical forms and facts are examples of metaphysical undefined terms and axioms.

Why conceal that ‘applying mathematics’ is a choice? It can be ‘rooting mathematics’, a phrasing indicating that ‘of course the roots should be treated before the plant’, mathematics. Or by saying ‘applying mathematics’ it can seduce teachers and students to believe that ‘of course mathematics must be taught and learned before it can be applied’.

Why conceal that ‘mathematics’ in Greek means knowledge that can be used for prediction? And why conceal that the predicting ability of calculations can be observed all over mathematics? Thus the calculation $3+4 = 7$ predicts the result of counting on 4 times from 3; and $3*4 = 12$ predicts the result of adding 3 4 times; and $3^4 = 81$ predicts the result of multiplying with 3 4 times. Likewise $6-3$ predicts the answer to the question $x+3 = 6$; and $6/3$ predicts the answer to the question $x*3 = 6$; and $3\sqrt{6}$ predicts the answer to the question $x^3 = 6$; and $\log_3(6)$ predicts the answer to the question $3^x = 6$. And why conceal that the predicting ability of mathematics made physics replace patronization with enlightenment?

Why conceal that mathematics can’t be well-proven statements about well-defined concepts after Gödel proved that not all statements can be proved; and Russell proved that using sets leads to internal self-contradiction: If $M = \{A \mid A \notin A\}$ then $M \in M \Leftrightarrow M \notin M$.

CONCLUSION

A teacher is paid by an institution to influence students in a way prescribed by its curriculum. In his book 'Modernity and the Holocaust' Bauman points to three conditions, singly or together being able to produce both a welfare society and a Holocaust: authorization, routinization and dehumanization (Bauman 1989: 21). Also he points out that the first two 'have been spelled out repeatedly in those principles of rational action that have been given universal application by the most representative institutions of modern society'. Especially the education institution enforces authorization and routinization by using curricula and exams and special teacher education. Thus the question arises: How can we know if this authorization and routinization leads to cognitive welfare or to a cognitive Holocaust?

Being solidly based on the sophist idea of enlightening nature in order to tell choice from nature, enlightenment institutions have a strong defense: Nature is there and needs to be enlightened to protect democracy. Being created by Humboldt as counter-enlightenment, Bildung institutions have big problems legitimizing their authorization and routinization. They only refer to a Bildung-discourse, referring on to a Didactics-discourse for its content.

Additional problems occur when using oral marks, becoming unreliable when given by the person also giving the teaching. In Denmark the use of oral marks in mathematics has forced the authorities to lower the passing mark in written mathematics from the international level at 60% correctness to 40% in the gymnasium and to 20% in the secondary school. When students can pass an exam by solving correctly only 1 problem of 5, it seems to indicate that the teaching has made the students not cognitive enlightened, but cognitive stunted.

The Nuremberg process in 1946 tried the German leaders for war crimes. However, they all excused themselves for having just followed orders. Only Keitel changed his meaning when enlightened about the Holocaust. To prevent a new process against teachers at pastoral Bildung schools in 200 years it might be a good idea if the authorization and routinization in mathematics education come from nature, and not from choices becoming pastoral by concealing other possible choices. Thus in the case of mathematics, set-based top-down metamatics from above should be replaced by multiplicity-based bottom-up mathematics from below respecting mathematics as a natural science investigating the natural fact many.

The educational institution conceals choices at all levels. The choices concealed are revealed when comparing the international standard set by the North American enlightenment schools wanting as many as possible to be educated as much as possible, with the EU Humboldt Bildung schools wanting only the elite to be educated for offices. The teachers are the interface between the institution and the humans forced to attend it for several years. Are teachers expected just to follow the authorization and routinization of the institution paying their wages? Or should teachers also be enlightened about the many concealed choices made by this institution? The survival of the EU population depends upon this choice.

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