

The Evaluation of Curriculum  
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“Curriculum” has multiple meanings related to the education of students. In our working group sessions on the evaluation of curriculum, we will focus on the two most prevalent meanings of curriculum: 1) curriculum as the set of learning expectations set by those in control of school policy at the country, state or local level, and 2) curriculum as the set of text materials used to guide instruction in classrooms. There are interesting issues related to each interpretation and issues at the intersection of the two interpretations. The creators of learning expectations for students and the developers of text materials for students consider each of these curriculum arenas. To help prepare for our working group discussion, we give a quick picture of developments in the United States over the past two decades in the evaluation of curriculum for each of its meanings.

The United States is a nation that has been distinguished in the past for the local control given to states and school districts to establish expectations and choose materials for educating students in mathematics. As the population became more mobile and as communication across states around the education of students increased, the great differences in what students experienced in schools in different states became more apparent. In the mid 1980’s, the National Council of Teachers of Mathematics (NCTM) made the bold move of appointing a group to write the first truly national set of standards for what students should be learning in mathematics in each of the grade bands -- 5 to 10 years-old (elementary), 11 to 13 years-old, (middle) and 14 to 18 years-old (secondary). The work of this group was published by the NCTM as the *Curriculum and Evaluation Standards for School Mathematics, K-12* (NCTM, 1991). To give you a flavor of the essence of this work, we include the titles of the mathematical areas for which expectations were written for each of the grade bands.

Each of the grade bands had four standards with identical names: *Mathematics as Problem Solving*, *Mathematics as Communication*, *Mathematics as Reasoning*, and *Mathematics as Connections*. These standards brought a different emphasis to the mathematical expectations for students’ learning that has persisted in much of the more recent standards setting efforts. These so called “process standards” were aimed at creating opportunities for students to see mathematics not as a set of isolated skills or facts to be learned, but as a human enterprise in which what has been learned allows us to think and reason in more powerful ways, to use mathematics to make sense of new and unfamiliar situations, and to make sense of mathematics as a whole by seeing its interrelationships—its connectedness.

At each grade band level the remaining standards were related to the mathematics content to be learned. At the elementary level the standards were labeled: Estimation, Number Sense and Numeration, Concepts of Whole Number Operations, Whole Number Computation, Geometry and Spatial Sense, Measurement, Statistics and Probability, Fractions and Decimals, and Patterns and Relationships. At the middle level the standards were: Number and Number Relationships, Number Systems and Number Theory, Computation and Estimation, Patterns and Functions, Algebra, Statistics, Probability, Geometry, and Measurement. At the secondary the standards were: Algebra, Functions, Geometry from a Synthetic Perspective, Geometry from an Algebraic Perspective, Trigonometry, Statistics, Probability, Discrete Mathematics, Conceptual Underpinnings

of Calculus, and Mathematical Structure. I expect that these labels resonate with labels that are used in other countries to indicate areas of mathematics that are important in K-12. Related to our work together, the last section of these 1989 standards entitled *Evaluation Standards* (NCTM, 1991) raises questions central to our task—evaluation of curriculum expectations, and possibly, the evaluation of how the expectations are or are not promoting the teaching and learning of the expectations (Standards). We begin with suggestions for looking at programs for their fit with standards.

The indicators for mathematics program evaluation given in the 1989 Curriculum Standards for School Mathematics document are the following (NCTM, 1991):

- Consistence with student outcomes; program expectations and support; equity for all students; and curriculum review and change.
  - Indicators of the Program’s match to the Standards should be collected in the areas of curriculum, instructional resources, and forms of instruction.
- Consistence with the standards.
  - The examination of curriculum and instructional resources should focus on goals, objectives, mathematical content; relative emphasis of various topics and processes and their relationships; instructional approaches and activities; articulation across the grades; assessment methods and instruments; and available technological tools and support materials.
- Attention to the mathematical content and its treatment.
  - Examine the relative emphasis assigned to various topics and processes and the relationships among them; opportunities to learn; instructional resources and classroom climate; assessment methods and instruments; and the articulation of instruction across the grades.

The following table from the 1989 Standards gives guidance in evaluating a program. “Program” includes learning goals, textbook materials, what happens in classroom lessons and student learning outcomes. Here again we see the interaction of evaluation of curriculum standards and curriculum materials as used in classrooms.

Table 3.1  
Purposes and Methods of Assessment (Part 5)

<b>Purpose (examples of questions asked)</b>	<b>For Whose Use</b>	<b>Unit of Assessment</b>	<b>Type of Assessment</b>	<b>Assessment Methods</b>
Program Evaluation • How effective is this instructional program in achieving our goals for mathematical learning?	Teachers  Administrators  Other decision makers	Class  School	• Tasks that reflect the intent of the curriculum goals  • Tasks that are aligned to the instructional	• Student interviews  • Performance tests  • Observation of class discussions

			methods and content of the curriculum (see Standards 12 and 13)  • Matrix-sampling test situations	• Success of students who have completed the program
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While some of these ideas more clearly belong to the discussion of evaluation of teaching, it seems appropriate here to look across a program to see the interaction of curriculum and its enactment. In 2002, the “No Child Left Behind” act was passed by the US Federal Government. This act required every state to create grade level learning expectation for grades K-8. The high school frameworks were articulated as either single subject or integrated courses.

The Center for the Study of Mathematics Curriculum (CSMC) funded by the National Science Foundation lead by Barbara Reys, Christian Hirsch and myself, has conducted a careful analysis of the grade level mathematics learning expectations (GLEs) for 42 states. Our analysis of the GLEs leads to another set of questions related to the examination of mathematics curriculum standards and materials. Our work is meant to challenge those who have responsibility for setting curriculum standards to produce standards that have the quality and clarity that can stimulate the development of excellent mathematics programs in schools. The concerns that surfaced in our analyses and our suggestions are relevant to our discussion.

### **Variation Across States**

We found great variation in aspects of GLEs set by the States. The first of these is the great variation in the kind of direction given to the users in the individual GLEs.

#### Specificity

The following GLEs illustrate the differences in the specificity across three states—Arizona, Colorado and Maryland-- in the area of functions (Reys, 2006, p. 69). In the case of Arizona, exactly the same GLE is given for each grade level from 4 to 8. With respect to Colorado, the GLE for grades 5 to 8 is nearly the same, the difference being in the kind of numbers considered in the quantities with which the students work. The change in the kind of numbers shows a logical progression from whole numbers to rational numbers over the grade span. The Maryland GLEs for function over grades 3 to 8 shows much more of a development of the meaning and use of functions as student progress over the grade levels. These examples show a range of specificity in the guidance given to teachers on what is expected for students to learn over the elementary and middle school years about functions. The question raised is, “What level of specificity is best to give teachers guidance that results in students learning?”

<b>State</b>	<b>Learning Expectation (GLE)</b>
Arizona	<i>Describe the rule used in a simple grade-level appropriate function (e.g., T-chart, input/output model). (the same GLE for grades 4-8)</i>
Colorado	<i>In any functional relationship involving whole numbers and common proper fractions, describe how a change in one quantity affects the other. (grade 5)</i>
	<i>In any functional relationship involving positive rational numbers, describe how a change in one quantity affects the other. (grade 6)</i>
	<i>In any functional relationship involving positive rational numbers and integers, describe how a change in one quantity affects the other. (grade 7)</i>
	<i>In any functional relationship involving rational numbers, describe how a change in one quantity affects the other. (grade 8)</i>
Maryland	<i>Complete a function table using a given addition or subtraction rule. (grade 3)</i>
	<i>Complete a function table using a one operation (+, -, x, ÷ with no remainders) rule. Assessment limit: Use whole numbers (0-50). (grade 4)</i>
	<i>Complete a one operation (+, -, x, ÷ with no remainders) function table. Assessment limit: Use whole numbers or decimals with no more than two decimal places (0 – 200). (grade 5)</i>
	<i>Interpret and write a rule for a one operation (+, -, x, ÷) function table. Assessment limit: Use whole numbers or decimals with no more than two decimal places (0 – 10,000). (grade 6)</i>
	<i>Describe how a change in one variable in a linear function affects the other variable in a table of values. (grade 7)</i>
	<i>Determine whether functions are linear or nonlinear when represented in words, in a table, symbolically, or in a graph. Assessment limit: Use a graph to determine if a function is linear or nonlinear. (grade 8)</i>

### Complexity

Another discernable difference in the GLEs across states is in the complexity of a standards statement itself. You will see in the following examples one state with a very dense and complex statement about the mathematics of expressions and equations and in a second example seven statements for grade eight that covers somewhat the same mathematical ground. The question to consider is, What balance of these two extremes gives the best guidance to those who use the GLEs?

<b>State</b>	<b>Learning Expectation (GLE)</b>
NH, RI, VT	<i>Demonstrates conceptual understanding of equality by showing equivalence between two expressions (expressions consistent with the parameters of the left- and right-hand sides of the equations being solved at this grade level) using models or different representations of the expressions, solving formulas for a variable requiring one transformation (e.g. <math>d=rt</math>; <math>d/r=t</math>); by solving multi-step linear equations with integer coefficients; by showing that two expressions are not equivalent by applying commutative, associative, or distributive properties, order of operations, or substitution; and by informally solving problems involving systems of equations in a context. (grade 8)</i>
	<i>Use physical models to add and subtract monomials and polynomials, and to multiply a polynomial by a monomial. (grade 8)</i>
	<i>Use symbolic algebra (equations and inequalities), graphs and tables to represent situations and solve problems. (grade 8)</i>
	<i>Write, simplify and evaluate algebraic expressions (including formulas) to</i>

OH	<i>generalize situations and solve problems. (grade 8)</i>
	<i>Solve linear equations and inequalities graphically, symbolically and using technology. (grade 8)</i>
	<i>Solve 2 by 2 systems of linear equations graphically and by simple substitution. (grade 8)</i>
	<i>Interpret the meaning of the solution of a 2 by 2 system of equations; i.e. point, line, no solution. (grade 8)</i>
	<i>Solve simple quadratic equations graphically; e.g., <math>y=x^2-16</math>. (grade 8)</i>

Our first two sets of examples have focused on the variety of ways in which GLEs are written relative to the level of specificity and the complexity of the GLE statements themselves. We also find variation among states in the use of language, verbs in particular, and in the level of cognitive demand suggested in the GLEs themselves.

### Cognitive Demand

Another area of concern for the researchers was the link between the demands of high school geometry and the geometry expectations exiting grade eight. The five levels of cognitive demand described by the van Hiele Theory (van Hiele, 1959) were used to examine the GLEs. The table below shows that the K-8 GLEs leave students far short of being able to explain and justify rules which seem critical for high school geometry. There was variation among the states, but as the table shows, few states contained GLEs calling for an explanation or justification of rules that would be the basis for reasoning and proof in high school geometry. The final area of variation among state GLEs is the grade placement of expectations. The findings here were reported in Volume 1 for Number and Algebra and in the draft of Volume 2 for Measurement, Geometry, Probability, and Statistics. (Smith & Tarr, in preparation.)

<b>Grade Level</b>	<b>Express/ describe a rule</b>	<b>Understand and Apply Rules</b>	<b>Analyze Rules</b>	<b>Explain/ Justify Rules</b>	<b>Total</b>
<b>K</b>	6	1	0	0	<b>7</b>
<b>1</b>	6	0	0	1	<b>7</b>
<b>2</b>	17	4	0	0	<b>21</b>
<b>3</b>	20	10	1	0	<b>31</b>
<b>4</b>	24	8	2	2	<b>36</b>
<b>5</b>	26	11	0	2	<b>39</b>
<b>6</b>	20	4	3	0	<b>27</b>
<b>7</b>	15	3	3	0	<b>21</b>
<b>8</b>	19	7	1	0	<b>27</b>
<b>Total</b>	<b>153</b>	<b>48</b>	<b>10</b>	<b>5</b>	<b>216</b>

### Grade Placement

What we see that is consistent across nearly all content expectations examined is great variation among the 42 states in the grade level at which a learning expectation is first introduced and the grade levels at which that learning expectation culminates. The chart below gives a typical example of what we found in most all the areas of content. If you

focus on the red triangles you see the variation in the grade levels at which the concept or procedure is first introduced. The blue squares note the level at which students are expected to have solidified their understanding of the concept or procedure in particular states.

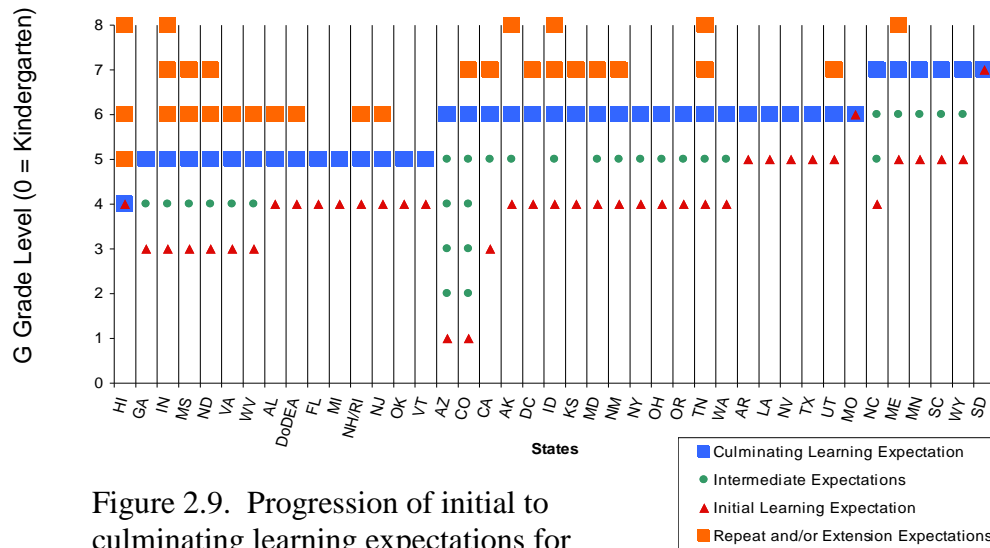


Figure 2.9. Progression of initial to culminating learning expectations for addition and subtraction of fractions, by state.

Two significant consequences of the variation among states in the GLEs are:

- The challenges of such variation for developers of textbooks, and
- The challenge of interpreting student learning across states or regions of the country.

### Recommendations

The two volumes of work on analyses of state grade level learning expectations conducted by CSMC have led us to make the following two recommendations:

- Identify major mathematical goals and build learning trajectories for these goals within and across appropriate grade levels.
- Collaborate to promote clarity and consensus.

The first of these recommendations makes a plea for careful examination of the growth of mathematical ideas in the GLEs over contiguous grade levels. What are the ways in which children are to engage with a mathematical idea over several grade levels as shown in the GLEs? Does the underlying mathematical idea become more sophisticated in a systematic way over time? Along what dimensions—size and kind of numbers involved, complexity of the problem situations through which the mathematics is evolving,

connections to other mathematics ideas, or in the growth of cognitive demand along thinking, reasoning, problem solving, and mathematical argument dimensions? This kind of careful development work on GLEs can lead to more discussion among teachers across grade levels around the development of the mathematics of the GLEs. This can have very positive consequences for teachers and their students.

The boundaries between countries and within countries are more permeable than ever. We will all be helped by collaboration to seek the best of educational innovation in the teaching and learning of mathematics and to use these ideas to create next generations of Mathematics Learning Expectations.

### **Evaluation of Curriculum**

Our major task in our time together is to examine ways of evaluating curriculum. While this paper has not taken a stance on what an evaluation plan *should be*, the paper has given a US report on past and current activities in setting learning expectations and has suggested some lenses through which to examine such a set of expectations and curriculum materials designed to enact those expectations. This paper is meant to start a discussion, from the participant countries perspectives, on the following:

- What is the role of curriculum expectations?
- Who is involved in setting expectations?
- For whom are such documents written?
- What are the consequences of such documents?
- What evaluation plan does each country have for judging a set of expectations?
- How is the decision to change a set of expectations made?
- How are mathematics text materials developed to reflect the expectations?
- Who is involved in the development?
- What help do teachers have or need to enact such materials?
- What kinds of evaluations can help continually improve both learning expectations and their related textbook materials?

We look forward to a productive discussion over our time together.

## References

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