

THE ROLE OF PHILOSOPHY IN MATHEMATICS EDUCATION

SOME EXAMPLES

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From 1997 we have created in Bergamo a “Workshop on Mathematics and Philosophy” with the aim to reflect about the role of philosophy in mathematics education, by referring to concrete classroom activities. In this contribution we would like to make some observations about three different activities we are proposing in classrooms: the first one concerns the paper folding and is carried out in classes with students from eleven to fourteen years old; the second one concerns the early algebra and is carried out in primary classes; the third one concerns the principle of relativity in Dante and is carried out in the last three years of high school.

PAPER FOLDING – DEMONSTRATION AND SCIENTIFIC ARGUMENTATION

When, between the end of seventh century and the start of sixth century B.C., in Greek colonies of Ionia, a new form of knowledge is born, that has revealed itself as a new way to link various knowledge among them. A theoretic aptitude was born from this form of knowledge, the capability of finding an unifying origin in infinite diversity of reality, searching a logical concatenation of diversity with “arché”. In this process, demonstration took shape, finding its formal configuration thanks to mathematics. [Riva]

Demonstration is the ground of western rationality, and its teaching is the main goal of mathematics. Our reflection concerns the didactics of demonstration. The difficulties of teaching demonstration are well known: there are difficulties of different degree, that can take back to different complexities of single demonstrations; but the fundamental obstacle is found in introducing fourteen - fifteen years old students to demonstration, because didactic experience very difficultly succeeds in rising in them the demand of demonstrating [Balacheff]. Teacher can ignore this fact and propose, from the very beginning, the demonstrative structure, directly referred to the system of axioms that justifies it; in this way he takes for granted that many students would unable to follow the proposed path. Otherwise, teacher can undertake a path of organisation of explorative activities in order to make them propaedeutic to demonstration; to this aim, to centre this way upon the argumentation seems to be the most effective instrument. In this case, the question of the relation between the didactics of argumentation and the didactics of demonstration arises. This question is matter of debate between who, as Balacheff, maintains that the relation is conflictual, and who instead, as Boero, maintains that

exists a continuity that link them and may have a didactic effectiveness. We believe that the continuity between argumentation and demonstration is better conceivable if “scientific” argumentation is taught, the argumentation that, first, was sublimated in Galileo’s texts and takes form and rhythm by acts; inference rules are directly referred to acts, that acquire the function of Galilean “sensate esperienze”.

The classroom activities we now briefly present, aim to introduce students to scientific argumentation. These activities deal with the problems of axes and angular bisectors in a quadrilateral [Locatelli]. These simple problems may become meaningful if managed by teacher in a suitable way. After the problem of axes and angular bisectors in triangles has been faced, teacher first of all, put the question if axes and bisectors of a quadrilateral are concurrent too; then, after the answers have been found, he asks when axes and angular bisector are concurrent. The specificity of these activities consists in the fact that the usual, quick, demonstrative way leaves place to the slowness of a proceeding in which the exploration by a program of dynamic geometry is joined with the concrete gestures of paper folding. The same slowness must be carried in the descriptive – argumentative structure that organizes results upon accomplished acts and gestures. Just acts, by their specific time organization, dictate the rhythm of argumentation, while the temporality of gestural representation permits to disclose and to recover modalities of thinking, according to proper times of learning by students. The shift from the practice of scientific argumentation, that finds its essence in referring to acts, to the demonstration, that exists because referred to a system of axioms, could be realized through a way in which the resort to historical and philosophical reflections acquires a very meaningful role.

The contraposition between the direct approach to teaching demonstration and the approach through scientific argumentation, mirrors an opposition that characterized western philosophical thinking since its origins: the opposition between “totalization” and “uni-versality”. The search of an unifying deductive chain that should coordinate all elementary geometry, engaged for centuries the Greek thinking, in a very open debate, which, among other subjects, involved even the problem if the sum of angles in a triangle is major, equal or minor of two right angles [Toth]. The admirable Euclidean construction acquired a totalizing character to the extent that, thanks to its exceptional functionality, broken off preceding debate, by imposing along millennia, a unique way of inquiry.

Same opposition seems burden today the choice of path in approaching didactics: to start from axiomatic structure, reflects a “totalizing” conception in cause of the fact that it reveals the awareness of possessed geometric “truth”, that students are obliged to accept as necessary. To construct a path of discovery that uses acts to organize scientific argumentation, means a choice of “uni-versality”, of a movement toward a not possessed unity; a path that appears more open to the “other”, to the “different”, because it can bring contributions and wealth too.

EARLY ALGEBRA – CORPOREITY AND EMBODIMENT

The second example refers to teaching of algebra in primary school. This problem rises a growing interest in research and practice of didactics of mathematics. Our working group since 1998 devotes itself to organization of activities, finalized to the construction of concept of quantity in children of primary school. [Bonetto] To this aim we have referred, above all, to experimentation that Davydov's group implemented in Moscow during sixties of twentieth century [Davydov]. Of this experimentation we have safeguarded the following very meaningful feature: the correlation between the introduction of algebraic code and the changes of quantities. Upon this feature we structured classroom activities, in which algebraic texts are "interpreted, controlled, justified, modified" [P.G. Ferrari].

The organization of activities has observed the following guidelines. We have conserved a central role to the "mental" register, that is to directly operate on algebraic relation, without external supplies. This register has been matched with different other register of representation: a concrete register, an iconic register, a numeric register and a narrative register.

Once the representations of "mental" are given, the problem is to voice them. So we have introduced to "communicative" register, integrated with description, argumentation and gesture. Gesture acquires a decisive role in order to dictate the rhythm of description with its integrative argumentation.

A reflection about the meaning of these activities meets a philosophical debate that could mark in a critical way the development of didactic activities. As our didactical proposal deals with the mental activity upon the algebraic symbols, it meets the Cartesian dualism between mind and body. This dualism seems now be overcome, but two very irreconcilable trends face each other: the one, that is summarized by the theory of embodiment, depicts the embodied mind, a mind basically rooted in the body, as a construction of metaphoric types that make mathematics to be [Lakoff]. In our opinion, this theory, differently from intentions of who propose it, doesn't break off with mechanism, because the metaphors take to simple abstraction and construction of conceptual artefacts. The other trend follows a reflections about the concept of corporeity in which the symbolic dimension is preserved; thanks to this fact it exceeds the simple concept and opens to a "further", that is pointed to, but that isn't conceptualized [Ricoeur]. This second trend seems now not too much involve the didactics of mathematics.

The two trends give birth to different languages and seem to indicate, in a sense, opposed didactic approaches. Just the activity about early algebra helps to make clear some diversities between the two approaches. The constructivist approach of embodiment reflects some its specificities in used language; its language insists on the needs to give meaning to algebraic symbols, it identifies this process with the term "objectification", it reduces the instruments to "artefacts" [Radford].

The approach that refers to corporeity inclines the language in another direction. It isn't necessary to give meaning to algebraic symbols; "they are history of activities". The "subject", with the fullness of his corporeity, is the maker of algebraic code, by interaction of acts; history of acts is seed for successive signifying. Rather than artefacts, in this approach, the various registers are different forms in which corporeity coordinates the "game" of algebraic code.

RELATIVITY IN DANTE – POETRY AND SCIENCE

In April 2005, the magazine Nature published a letter by Leonardo Ricci, physician near the University of Trento, in which author maintains that in some tercets of Dante's Hell there is an exact description of the principle of relativity. The fact is surprising because the Divine Comedy was written in the first part of fourteenth century, while the principle of relativity was introduced by Giordano Bruno, in *La cena de le Ceneri* (1584), and was clearly fixed by Galileo, in *Dialogo sopra i due Massimi Sistemi* (1632).

This news suggested us an activity that we have carried out in classroom. We have started by making clear some key aspects of the principle of relativity, that is, the determination of the state of rest, the inertial frame, the relativity of motion [Landau]. Then we have faced the Dante's text, with the aim to make explicit which aspects of principle of relativity has been perceived by Dante's intuition, and to discover in which way Dante conveyed them in his poetry.

The involved canto is the seventeenth of the Hell "*nel quale si tratta del discendimento nel luogo detto Malebolge, che è l'ottavo cerchio de l'inferno; ... il demonio Gerione sopra 'l quale passaro il fiume...*" (in which it is dealt of the descent to the site, that is called Malebolge, the eighth circle of the Hell ...; they got over the river on the back of demon Gerione...")

First of all, Dante has perceived the demand to put himself in a state of rest. To this aim is necessary to assume that "the system" is as much as possible far away from everything that can have influences on it. In the following lines, Dante seems to have percept the importance to put himself in a system free from external influences:

<i>Maggior paura non credo che fosse</i>	<i>Not greater was the dread</i>
<i>...</i>	<i>than was my dread, when round me on each</i>
<i>che fu la mia, quando vidi ch'i' era</i>	<i>part</i>
<i>ne l'aere d'ogne parte, e vidi spenta</i>	<i>the air I view'd, and other object none</i>
<i>ogne veduta fuor che de la fera.</i>	<i>save the fell beast.</i>

Here Dante translated into its poetic language the search of a reference that, at his time, was at the centre of process of geometrisation of space that his contemporary artists, as Giotto and Lorenzetti, were pursuing.

The other two intuitions about the principle of relativity are expressed in the following tercet:

<i>Ella sen va notando lenta lenta; rota e discende, ma non me n'accorgo se non che al viso e di sotto mi venta.</i>	<i>He slowly sailing, wheels his downward motion, unobserv'd of me, but that the wind, arising to my face, breathes on me from below.</i>
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The slowness that is solemnly exposed in the first line, suggests the intuition of a system that, today, should say nearly – inertial; in it, just thanks to extreme slowness of motion, the effects of rotation are made negligible.

In “ma non me n'accorgo” (unobserv'd of me) there are implicitly contained Galilean conclusions that a) the phenomena that happen “when the ship is motionless” and the ones that happen “when the ship is in motion” are described by same physical laws, and b) it is in no way possible to decide, by observing the phenomena “below deck, if the ship is motionless or in motion”. In “non me ne accorgo” (unobserv'd of me) there is a first perception of the fact that “the state of rest isn't a favoured state”.

A very interesting aspect of that tercet stays in its “stylistic” structure: Dante uses different linguistic registers to express different sensations.

The first line is aulic and, thanks to prevalent use of “a” and “en”, it transmits the idea of tranquillity, that should be experienced in a free from external influences system.

The second line is broken: the central “ma” introduces a sharp split and expresses the sense of astonishment that Dante feels in front to his intuition.

The third line is less harmonious and seems to translate a certain trouble in cause of the fact that wind hinders to arrive to full tranquillity.

The philosophical reflection that arises about this activity concerns the general question of two cultures, that is, “the distinction between scientific and humanistic culture, that is so rooted in our current lexicon, in scholastic partitions, in studying experience of each well-cultured person”. [Sermonti].

According to Giovanbattista Vico “the most sublime work of poetry is to give sense and passion to senseless things... Poetic science concerns imaginative capabilities” and poetic characters are images and metaphors of what impresses the senses. To this science of narrative or imaginative understanding, that is obtained by fantasies of mind, the abstract system of Cartesian science opposes. An intense work of research has been made to recover for the abstract system of science its originary link with language: “Mathematics is daughter of language... Language is the authentic and primitive way of exchange” [Prodi].

The extraordinary feature of Dante's poetry stays in the fact that it unveils a rather neglected side of ratio between the two cultures. "Dante is a scientific poet, he's a poet because a scientist, he's a scientist because a poet... His poetic language is available for all adventures of knowledge, included the careful observation of clinical, optical or astronomical phenomena, ... included the mathematical structure that transforms itself in proportion and "convenienza" (from Latin "con- venire", *to agree, to enter in consonance*) between world and the vision of world» [Sermonti].

In Dante's text there is a relation of mutuality between language and science: science is daughter of language but language can take form from science. His resorting to allegory frees his text from the usual link among object, meaning and signifying image. The "arbitrary and intentional" that are proper of allegory, allow Dante to evoke those intuitions to which "his extraordinary mental autonomy, his omnivorous eclecticism, his anachronism" [Sermonti] carry him. These intuitions, sometimes, aren't "imaginative fantasy" but are strongly scientific; their interpretation need of a context, "of a rational and intellectual process, susceptible of the critical discussion".

CONCLUSIONS

In the upper reflections we have dealt with the contraposition between totalization and "uni-versality", between embodiment and corporeity, and with the problem of the two cultures. These themes enter in the unitary reflection that has been developed in our workshop on mathematics and philosophy along all its path. At the centre of our reflection there is the theme of identity. On the one side, this theme is today characterized by demands of defence and by answers of closure that creates forms of truth based on "totalization". In these latter everyone opposes his truth to the one of other. On the other side, this theme singles out an attitude in which the deep meaning of identity is replaced by "generalizations, to which we shouldn't attach too much" (Spagnolo), because they are constructions that have lost any symbolic dimension.

Dante seems to overcome these two positions and to suggest a synthesis that today can still be worth. Research of identity comes true, in Dante, through his symbiosis with the originary; in Dante that process of formation of western rationality it is sublimed, where "imagination is structured according the sense of harmony and proportion, where art of motivation is constant, where reality is presented in its entirety" (Reale). Dante's mental autonomy, his eclecticism, his anachronism, confer to his poem a strong sense of opening, an "exceeding towards the further", in which poetry and science convey a great passion.

Didactics needs of this consent between research of originary and opening to new; originary is intended as rediscovery of one's culture and improvement of corporeity and of its rhythms of learning; opening to new is intended as a didactics in which the corporeity, in its plenitude, opens to the "other", in an attitude of sharing, of research, of "delicatezza" (Longoni).

References

- Ball, D. L. (1990). Prospective elementary and secondary teachers' understanding of division. *Journal for Research in Mathematics Education*, 21(2), 132-144.
- Balacheff N. (1999, May/June) Is argumentation an obstacle? *International Newsletter on the Teaching and Learning of Mathematical Proof*.
- Boero P. (1999, July/August) Argumentation and mathematical Proof. A complex, productive , unavoidable relation in mathematics and mathematics education. *International Newsletter on the Teaching and Learning of Mathematical Proof*.
- Bonetto M., Bonissoni P., Soffientini D., Rottoli E. (2007) Algebra per la scuola elementare: il simbolo algebrico è storia delle attività. *Booklet; in preparation*.
- Davydov V.V. (1982). The psychological characteristics of the formation of elementary mathematical operations in children. In T.P. Carpenter, I.M. Moser & T.A. Romberg (Eds.), *Addition and Subtraction: A cognitive perspective* (pp. 224-238). Hillsdale, NJ: Lawrence Erlbaum.
- Ferrari P.L. (2004). Tecnologia informatica e sistemi di rappresentazione nell'insegnamento universitario della matematica. www.mfn.unipmn.it.
- Lakoff G., Nunez R.E. (2000). Where Mathematics Comes From : How the Embodied Mind Bring Mathematics into Being. *Basic Books*.
- Landau L., Rumer Yu. (1979) What is the theory of relativity? *Mir Publishers Moscow*.
- Longoni P., Riva G., Rottoli E. (2001) Ethics of "delicatezza" *CIAEM 53. Mathematical literacy in digital era. Edited by L.Bazzini, C.Whybrow Inchley. Ghisetti Corvi Editori. Pag.291*
- Locatelli M., Rottoli E. (2008) Pensare con le mani - Le piegature della carta. *To be published*.
- Prodi G. (1977) Le basi materiali della significazione. *Casa editrice Bompiani. Milan*
- Radford L. (2006) Elements of a cultural theory of Objectification. *Revista Latinoamericana de Investigation en matematica Educativa. Pp 103-129*.
- Ricoeur P. (1997) La metafora viva. *Jaca Book, Milano*.
- Reale G. (1989) Storia della filosofia antica. *Vita e Pensiero, Milano*
- Riva G., Rottoli E. (1998) Geometrie non Euclidee: Matematica, Storia e Filosofia. *Provveditorato agli Studi di Bergamo*.
- Sermonti G. (2003) Con Dante, oltre le "due culture". *Il Sole 24 Ore. 19 Ottobre*.
- Spagnolo F. (2008) Alcune idee sulla Filosofia dell'Educazione Matematica tra oriente ed occidente. <http://math.unipa.it/~grim>.
- Toth, I. (1997) Aristotele e i fondamenti assiomatici della geometria. *Vita e Pensiero. Milano*