FIVE CHALLENGES TO INSTRUMENTAL GENESIS

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Abstract. Opportunities to develop and apply mathematics appear in modern society through instrumentation and instrumentalization. The first one means that technology shapes the actions of the users, whereas the latter refers to the fact that also the mathematical objects to be investigated are shaped by the users. Mathematicians and educators on all levels must be able to master this genesis within instrumental orchestration. Research should produce sustainable and viable frameworks for teaching praxis so that teachers would be able to scaffold collaborative constructions process of viable knowledge through radical constructions among the learners.

Introduction

In a modern society, technology has caused a holistic shift in way we think, plan and evaluate. This development together with a changed conception of knowledge and learning could lead to a paradigm shift: learning of mathematics is more distributive (i.e. independent of time, place and formal modes), socio-constructivist (learning community centred) and technologically enhanced (Haapasalo & Silfverberg 2007). This potential is used not only via networks or computers but also on calculators and communicators, which students use in informal way on their free time. When using a tool within more or less spontaneous procedural[1] actions, the tool, especially at the beginning, puts certain limitations on what can be investigated and how. Adapting the term of Trouche (2004), I mean by *instrumentation* the process when the tool shapes the actions of the users. On the other hand, users often find their own schemas and schemes to use the tool. In this process of *instrumentalization* not only the use of the tool, but also the objects to be investigated are shaped by the users. Today, any sophisticated user skips unnecessary manipulation of $1/\sqrt{2}$ to $\sqrt{2}/2$ by using the calculator keys SQR and 1/X. This instrumental genesis has an impact on how mathematical situations appear for a modern citizen. On the other hand, progressive technology can be interpreted as an orchestra. Therefore it seems appropriate to use Trouche's term *instrumental orchestration* when scaffolding student's instrumental genesis. The instrumentation of a sophisticated

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[1] I adopt the following characterizations of Haapasalo and Kadijevich (2000):

- *Procedural knowledge* denotes dynamic and successful use of specific rules, algorithms or procedures within relevant representational forms. This usually requires not only knowledge of the objects being used, but also knowledge of the format and syntax required for the representational system(s) expressing them.
- *Conceptual knowledge* denotes knowledge of particular networks and a skilful "drive" along them. The elements of these networks can be concepts, rules (algorithms, procedures, etc.), and even problems (a solved problem may introduce a new concept or rule) given in various representational forms.

Based on the logical relation between these knowledge types, two pedagogical approaches are defined: *developmental* and *educational.*

impersonal device to instrumentalized personal device is often a long process of instrumental genesis. This forces educators to re-consider the interaction between process and object features of mathematical knowledge. At the same time, to make problem-solving devices to pedagogical tools, intensive co-operation is needed among hardware and software designers and researchers of mathematical learning processes. To develop research-based frameworks for instruction design we need to combine knowledge of mathematics and its history, philosophy, psychology, sociology, physiology and *ICT,* for example. This article emphasizes four of the many challenges and discusses at the examples of pedagogical implementation with critical issues.

Challenge 1 - Solid framework theories for collaborative social constructions

Mathematics education research should produce sustainable and viable frameworks for teaching praxis so that teachers would be able to scaffold collaborative constructions process of viable knowledge through radical constructions among the learners.

Groundings: The slogan of constructivism in our scientific community started about twenty years ago. Later on this discussion extended to include collaborative social group processes. Still, there are quite few empirically tested models how teachers can plan and realize learning environments within that paradigm. Even though there are numerous studies on *collaboration* and *group development*[2], the impact of knowledge structure, pedagogical philosophy and support for reflective communication has been neglected.

With respect to our *socio-constructivist* framework, the famous *pragmatic theory of truth* emphasized by the famous philosopher Charles Peirce would give a solid basis, making the debate between radical and weak constructivism sound unnecessary and even naive. When an open problem is given, namely, the teams work in causal interaction with this problem under collaboration. After testing the viability of radical ideas among the teams and between the teams, only those ideas finally remain which are viable for the whole social group consisting of those teams (see Figure 1).

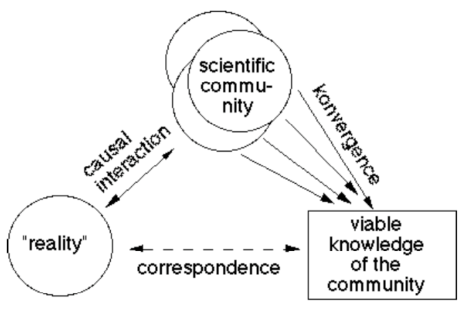

Figure 1. Viable knowledge as a result of radical social constructions (Eskelinen & Haapasalo 2007).

[2] When typing-in 'Tuckman group processess', for example, Google produces more than 72000 hits with interesting links, whilst for "collaboration" the are almost 100 million hits (retrieved 10th of Feb 2008).

Example of implementation: Eskelinen and Haapasalo (2006; 2007) suggest that design of technology-based learning environments within an adequate constructivist theory linked to the knowledge structure might be a proper framework to respond to the main challenge of teacher education: to get students understand which are the basic components of modern constructivist theories on teaching and maintaining the learning through cognitive conflicts. Their studies reveal that working within socio-constructivist collaborative *ICT*-based design processes for the production of a hypermedia-based learning environment, even during a short period of time, changed student teachers's conceptions of teaching and learning from an objectivist-behaviorist viewpoint to a constructivist view, and decreased students' interest in having support for computer routines.

Critical issues of implementation: To analyze very complicated studying environments several aspects should be considered at the same time. The research tradition in mathematics education community has, unfortunately, concentrated on focusing on one or few components at a time.

Challenge 2 - Solid framework theories for linking conceptual and procedural knowledge

The following two dilemmas have been neglected in our scientific community, even though they might appear throughout our mental life: (1) Do we have to understand being able to do, or vice versa? and (2) Should mathematical objects be emphasized as objects or as processes when learning them?

Groundings: Literature analysis reveals the dominance of procedural knowledge over conceptual one in the development of scientific and individual knowledge (Haapasalo & Kadijevich 2000, Zimmermann 2003). We know from the basics from cognitive psychology that *our world is a world of meanings, not a world of stimuli*. The investigation problems (labeled "reality" in Figure 1) should be psychologically meaningful for the students. This implies the need to apply the so-called *developmental approach* in the instructional design: students should have opportunities to go for their more or less spontaneous procedural knowledge. On the other hand, perhaps the most important educational goal in a modern society is – especially if we trust on mathematics' power to trigger general educational goals - to scaffold citizens' abilities to identify and construct links within complicated multi-causal and multi-disciplined knowledge networks. This means investing on conceptual knowledge, even in such a way, that students also learn appropriate procedural skills. This so-called *educational approach* causes the following conflict: *Does a student have to understand being able to do, or vice versa* (Haapasalo 2003). This dilemma is forced by the fact that for too many students, one of the basic difficulties for the learning of mathematics is that very often entities appear as objects as well as processes.

Example of implementation: The quasi-systematic framework of author's MODEM project[3] has been successfully tested for planning and assessment within many conceptual fields of school mathematics. It offers a sophisticated interplay between developmental and educational approach. When planning a constructivist approach to the mathematical concepts under consideration, the focus is on developmental approach. On the other hand, when offering students opportunities to construct links between representation forms of a specific concept, the focus is on educational approach, enhancing links between mathematical representations.

Critical issues of implementation: It seems that our community is contaminated with so-called "romantic constructivism" without any particular educational goals: students should be allowed to construct what they will. Hence, many researchers see that systematization contradicts the starting point of constructivism. Furthermore, quite poor metacognitive abilities among teachers and students can prevent utilizing the MODEM task types even though they would be tailored to learning with very simple actions (see Haapasalo 2003; 2007). Hence, the own design of both investigation tasks and tasks to link mathematical representations might be even more difficult to teachers.

Challenge 3 – Dilemma between systematic models and minimalist instruction

To emphasize the genesis of heuristic processes and students' ability to develop intuition and mathematical ideas within constructivist or minimalist approach a systematic planning of the learning environments (i.e instrumental orchestration, to be explained later) is needed. In learning situations, however, students must have freedom to choose the problems that they want to solve within continuous self-evaluation instead of relying on guidance by the teacher.

Groundings: Zimmermann's (2003) study of the history of mathematics reveals eight main activities, which proved to lead very often to new mathematical results at different times and in different cultures for more than 5000 years: *order, find, play, construct, apply, argue, evaluate, and calculate*. Especially the five first activities very often run optimally without any external instruction or demand. Students frequently neglect teacher's tutoring or they feel they do not have time to learn how to use technical tools. Teachers similarly feel they do not have time to teach how these tools should be used. This problem becomes even more severe when the versatility of advanced technology cannot be accessed without first reading heavy manuals. The term *minimalist instruction*, introduced by Carroll (1990), is crucial not only for teachers but also for those who write manuals and help menus for the software.

[3] see Haapasalo 2003; 2007 or http://www.joensuu.fi/lenni/modemeng.html

Example of implementation: MODEM-framework can be used for the planning and assessment in systematic way, whereas in authentic learning situations the framework stays on the background in quasi-systematic way. The following example of the *ClassPad project* (Eronen & Haapasalo 2006) shows that the challenge can be responded by organizing different task types into a "problem buffet". To go for linear function, one student team, for example, initially selected quite a complicated problem series on optimizing mobile phone costs. After realizing that the (partly linear) cost models appeared too difficult for them, they then chose a new, much easier, problem set, which happened to consist of identification tasks – the first and lowest level of the concept building within the systematic MODEM framework, which was on the basis of the planning of the learning environments. This example shows that a sophisticated interplay between a systematic approach and minimalism can be achieved even by simple pedagogical solutions.

Critical issues of implementation: To be able to compose theoretically grounded approaches in appropriate way, teachers must be deeply involved in problem-solving culture, which is not usually the case. Even though the final pedagogical solution might be simple, cheap and easy to use, teachers might have difficulties to combine those many aspects that are needed for this kind of task design.

Challenge 4 – Shaping the instrumental genesis through instrumental orchestration

The constructivist viewpoint states that both making of mathematics and teaching of mathematics must relate to the instrumental genesis in modern society.

Groundings: Instead of old-fashioned "papermedia", we can enrich curriculum by revitalizing curves, being topic for famous mathematicians and physicists of 17^{th} and 18^{th} century. Today they can, namely, be visualized even with pocket computer almost by any layman. Educators on all levels must have know-how for scaffolding this genesis, which has already changed - and it will change even more radically - our views on making and teaching mathematics. The fact that most part of students' instrumentation and instrumentalization very often happens on their free time, implies that educators should shift the focus from well-prepared classroom lessons on minimalism. Instead of acting like a pace car in a race, institutions should be types of pit stops to scaffold students' "race" outside the classroom. By looking the relationship between technology and mathematics education from five perspectives, I suggest that instead of speaking about 'implementing modern technology into classroom' it might be more appropriate to speak about 'adapting mathematics teaching to the needs of information technology in modern society' (Haapasalo (2007). This means emphasizing more the making of informal than formal mathematics within the framework of the above-mentioned eight main activities and motives, which have proved to be sustainable in the history of human thinking processes and making of mathematics (see Zimmermann 2003). The fact that ordinary people can realize outstanding examples of simple and powerful ideas from the history of mathematics implies that also organizing the content of the curriculum should be made in a meaningful way instead of treating the same idea in several disguised forms under the guise of “spiral curriculum”.

Examples of implementation: Prototypes of revitalizing geometric ideas from the history of mathematics can be found and downloaded from the websites referring to material production within our Joint European project[4]. Those visualizations can be utilized in many ways almost on any level of mathematics teaching. The first level of modelling could be just to watch the beautiful simulation and try to explain in own mother tongue what happens on the screen. The highest level of modelling would be to make an own computer-based model, which makes the same simulation or perhaps improves it.

Eronen & Haapasalo (2006) and Haapasalo (2007) illustrate a successful instrumentalization within the simultaneous activation of conceptual and procedural knowledge with the ClassPad calculator[5]. Even though this totally new tool was shortly represented to students just few days before their summer holiday aif they wanted, portfolios show that students moved from instrumentation to instrumentalization without any tutoring from teacher’s side.

Eronen & Haapasalo (2006) used Zimmermann’s idea to model mathematical activities as an octagon and to quantify each activity to find out mathematical profiles among teachers and students. The results suggest that doing mathematics with ClassPad, even during a short period of time outside the classroom, enlarged 8th grade student’s mathematical identity within these motives and activities. Student’s portfolios reveal sophisticated metacognitive skills[6] within their instrumentalization. Later on, the learning of mathematics at 9th grade only with ClassPad without any textbooks within interaction between minimalism and systematization was successful concerning students’ cognitive development. Students scored in all test items significantly better after the ClassPad working than in the pre-test. They also showed remarkable procedural skills not only connected to the linear function but to other function types. A postponed test, after 5 months, revealed that this scoring level remained consistent, and for many students it even improved. Students liked the feeling that they had reached action potential, which was described to be one of the main aspects in assessment within minimalism. They also liked the learning without any pre-set goals or tutoring from teacher’s side.

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[4] see http://www.joensuu.fi/lenni/modem.html; http://www.math.jyu.fi/~kahanpaa/TUBerlin/home.html
[5] see http//www.classpad.org
[6] Haapasalo (2007) reveals degenerated metacognitions among teachers and students when working with ICT in school.

Critical issues of implementation: Perhaps the most difficult obstacle in the instrumental orchestration is that educational policy makers in most countries are against allowing the use of modern technology in examinations. This implies that teachers are even less voluntary to learn to develop their own instrumental genesis, and even less ready to make the same concerning instrumental orchestration. As another problem I would like to mention the fact that usually technological tools are made for those who apply mathematics, and not for learning purpose. So, it is a very difficult task for us educators to shift this development in educational direction. Manuals, as for ClassPad for example, consist often several hundredths of pages containing huge amount of conceptual mathematical knowledge. This causes a contradiction between the versatility of the tool and minimalist instruction.

Challenge 5 – Applying business principles to shift the bad reputation of mathematics

Instead of talking about internal problems of mathematics, educators should discuss the issue as a managerial problem.

Groundings: When people critisize mathematics, they may mean the science itself or its teaching in schools. If would be an enterprise named ‘Teaching Mathematics’, it would have probably already crashed and ‘Mathematics’ would disappear from most schools like it was the case with the Latin language. Technology probably accelerates this process. From a managerial point of view, Hvorecky (2007) considers teaching Mathematics as a (virtual) company and suggests the following measures to be involved in educational research: (1) Each “market segment” has its own expectations. Thus, we should set up relevant priorities for different groups of pupils/students; (2) As “the customer is always right”, we should make mathematics more “edible and digestible” for each segment i.e. closer to their environment and cultural values.

Examples of implementation: There are numerous researches pointing out the publich image of mathematics. Again, when testing how eager the discussion is, I typed “public image of mathematics” in Google, getting more than two million hits. Educators should try to “clean all this mess” and pick up elements, which could lead to successful marketting ideas and new curriculum design. My own effort has been, for example, to combine vocational and comprehensive school sector by designing proble,-based self-determined learning materials[7].

Critical issues of implementation: Our scientific community is contaminated with a tradition to speak about internal problems of mathematics or its teaching.

[7] http://www.joensuu.fi/lenni/vocation.html

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