Potentials of Spatial Geometry Curriculum Development with Three-Dimensional Dynamic Geometry Software in Lower Secondary Mathematics

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The purpose of this research is to clarify the potentials of spatial geometry curriculum development in lower secondary schools through utilizing three-dimensional dynamic geometry software. By applying the impact of this software which is epistemological, five potentials are identified as followed: empowering the learning activities in spatial geometry, empowering activities wherein space figures are logically explored, expanding the learning content of spatial geometry, strengthening the connection of content in different units, strengthening the relationships between mathematics and real world.

1. The necessity for developing spatial geometry curriculum with 3DDGS

The space surrounding us has three dimensions, namely, “length,” “width” and "height”, for example. We live in this three-dimensional world. So, with mathematical “glasses” of spatial geometry, we can obtain a deeper understanding of the mechanism and scheme of the three-dimensional world, improve the quality of our daily lives, and perceive four or higher dimensional worlds which we cannot see in nature (Banchoff, 1990). By using three-dimensional figures in those ways, therefore, our activities as humans can be further enriched.

It is necessary for learners to appreciate the value and merit of learning spatial geometry as well as merely to learn properties and relationships in spatial geometry. To respond to that it is necessary for us to reflect a new approach to spatial geometry curriculum not only from aims and contents but also from a learning environment. Because developing curriculum based on an educationally effective ICT environment will support dealing with contents that was difficult to learn/teach in conventional environments, eliminating limitations of learning/teaching, and creating various potentials in their aims and assessments.

Regarding ICT environments in geometry, Dynamic Geometry Software (hereafter referred to as DGS) being arrived in the 1980s, have created new potentials of construction and operation of plane figures, and have enriched learning/teaching of plane geometry at the mandatory educational stages. In recent years, researches relevant to DGS have begun to focus on how DGS can help mediate explanations, verifications and the nature of proofs (Hoyles & Noss, 2003). Based on changes in learning environment and a trend in research, long-term curriculum development projects such as the Compu Math Project

(Hershkowitz et al., 2002) have been conducted at the lower secondary school level, and fruitful results achieved. However, conventional DGS has been mainly developed and used in the learning/teaching of plane geometry, so this DGS has had little influence on curriculum development in spatial geometry.

In response to this, three-Dimensional Dynamic Geometry Software (hereafter referred to as 3DDGS) appeared in 2004. When we use 3DDGS, similar to conventional DGS for plane geometry, we can construct spatial figures, operationally and manipulate space figures while maintaining their properties and relationships in a three-dimensional world. Moreover, we can observe simultaneously the movement of figures not only from various perspectives but also on different screens. If learners can use 3DDGS, it will facilitate to improve their learning of conventional contents. In addition, it enables developing new contents which was difficult to learn until now. Moreover, it can be expected that more active learning and teaching will be enhanced. Furthermore, the correspondence between space figures and our surroundings can be strengthened by using 3DDGS, therefore it will presumably foster “Mathematical literacy.”

2. Purpose and methods

The purpose of the research is to answer the following questions.

Through applying the epistemological impact of 3DDGS, what kind of potentials would be created in the curriculum development of spatial geometry in lower secondary schools?

3. The epistemological impact of 3DDGS

ICT in mathematical education such as DGS is a “window” at the boundary of the metaphysical world known as mathematics and the physical world in which we live. It is also a "stick" with which we can construct and manipulate mathematical objects and relationships for any purpose. Thus, researches relevant to ICT in mathematical education have developed in immanent realism (Resnik, 1992), so we in the physical world can contact, through ICT, mathematical objects and relationships in the metaphysical world.

In particular, ICT in mathematics education has an impact on refining how we recognize mathematical objects. Moreover, the new realism that substantializes mathematical objects and relationships has challenged potentially conventional premises of learning/teaching and has entailed deep changes in curriculum and learning/teaching (Balacheff & Kaput, 1996).

For example, ICT gives us substantiations for mathematical objects and relationships, and then helps us to manipulate and observe them. Therefore ICT promotes our conjectures and reasoning regarding how mathematical objects will behave in response to the various approaches taken. Thus we obtain a deeper understanding of mathematical objects and decrease the “distance” between mathematical objects and us. And as a result, we actually realize familiarity with a mathematical object as "something of our very own". We can also change the connection between people as the subject and a mathematical object as an object into something more fruitful. In this sense, it is worthy of mention that the impact of ICT in mathematics education is epistemological (Piaget, 1947).

In case of 3DDGS, the objects are constructed with spatial figures, so they correspond to the real world more compared to 2DDGS. Thus, with 3DDGS, it is easier for us to vertically examine spatial figures in mathematics and horizontally examine them in mathematics (Treffers, 1986). These examinations will encourage a disposition that do not merely replicate the real world with spatial figures but also attempt to get a grip of the real world.

This disposition also has epistemological meaning in the relationship between geometry and the real world. And that is because, in this disposition, it is possible to see the “Copernican Revolution” (Kondo, 1994) that occurred in the history of geometry philosophy, particularly while non-Euclidean geometry was being established, where a passive perspective in the sense that a single absolute space is “transcribed” into geometry was changed into an active perspective in the sense that geometry give the space a structure.

4. Epistemological impact on curriculum development using 3DDGS

4.1 Empowering the learning activities in spatial geometry

Applying the epistemological impact of 3DDGS enables constructing objects based on the properties and relations of the figures used. It also enables observing the process of constructing objects and then observing them as the result of construction. Subsequently, the epistemological impact of 3DDGS can open up more potentials of empowering the learning activities in spatial geometry.

For example, in the Japanese government educational curriculum guideline (revised in 1998), seventh grade learning content includes activities that aim at understanding that space figures such as cones and cylinders can be constructed by rotating a plane figure. In class teachers are likely to demonstrate constructing a space figure with teaching tools whereby a planar board is rotated around an axis. However, for students who do not realize that a space figure will be constructed through that rotation of a plane figure, it is rather difficult for them to visualize the space figure from the "afterimage" of that figure. Contrasting that, with 3DDGS, rotating a plane around an axis and tracing the perimeter is enabled (Figure 1). Moreover, when the plane is modified and then rotated, various solids of revolution can be constructed.

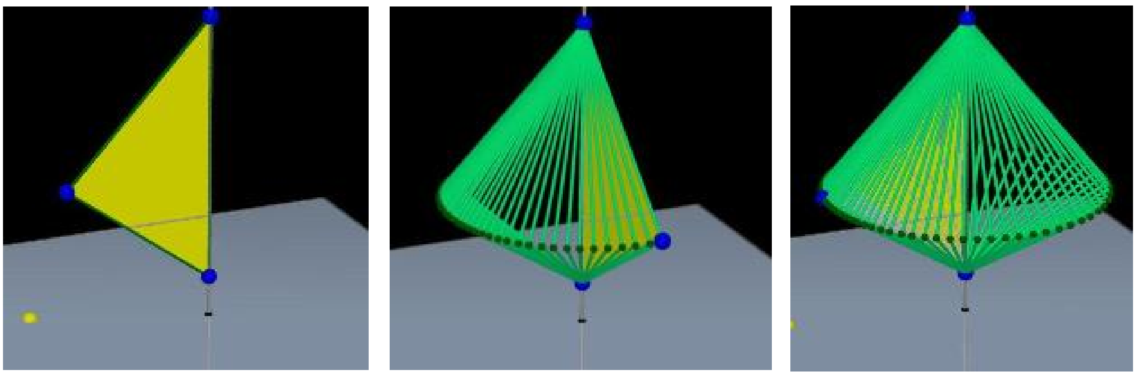


Fig. 1 Rotating a triangle and tracing the perimeter

4.2 Empowering activities wherein space figures are logically explored

Applying the epistemological impact of 3DDGS enables observing the construction and manipulation of space figures with dynamic transformations in multifaceted ways. It is considered that the above can improve both elaborateness and precision of observation, deepen relational understanding of objects in three-dimension, and amplify the quality of verification and discovery. Moreover, through reverse transformations whose attributes are maintained, the properties, relationships, dependence and independence of the elements that make up the objects become evident. This then further empowers activities wherein the properties of space figures can be logically explored based on the properties and relationships of plane geometry.

For example, in the process of exploring various cut surfaces of space figures, students discover that the cut surface sometimes becomes an equilateral triangle, a square (regular quadrilateral) or a regular hexagon. And at this point they can discuss exactly why the cut surface became, for example, an equilateral triangle, after a student has cut a cube with a kitchen knife in 3DDGS, they can then observe by moving the kitchen knife forwards and backwards within the cube that the cut is diagonal to the bottom surface when you start cutting the cube (Figure 2, left). We can also observe in the middle of cutting that the cut is diagonal to the side surface (Figure 2, right). Through these observations it can be considered that students would find it easier to explain, based on the fact that the individual surfaces of the cube are congruent, that "the surface cut is an equilateral triangle because the diagonals of the congruent squares are of equal length and because the triangle consists of line segments of equal length."

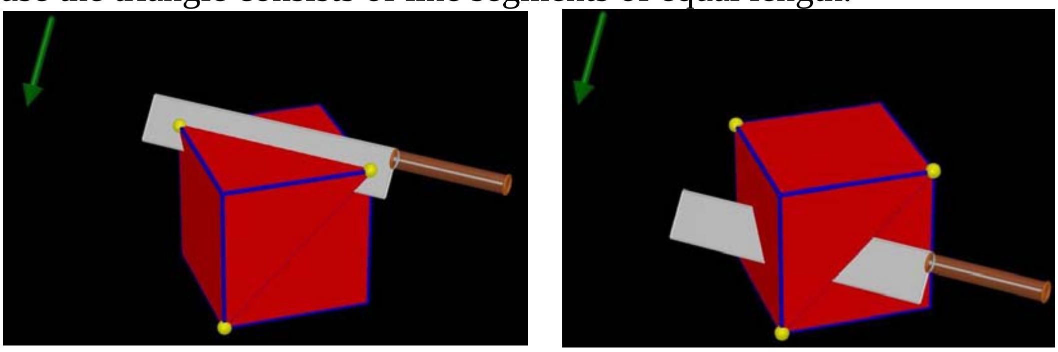


Fig. 2 Cutting a cube with a "kitchen knife"

The students continue exploring whether the surface cut will become a regular pentagon. When the sides or angles of surface cut are measured with the measurement function of 3DDGS, it is then possible to observe that the surface cut is not a right pentagon even though it sometimes appeared to be one on the 3DDGS screen. Moreover, when the kitchen knife is moved forwards and backwards within the cube, students can focus on the positional relationships of the surfaces cut. And in this way they can observe that at least one set of the sides of the cut surface pentagon are parallel because the corresponding surfaces of the cube are too. However, a right pentagon does not have a parallel set of sides, so the cube's cut surface cannot be a right pentagon. This exploratory activity becomes the basis for the learning of indirect proof because students have a chance to explain the contraposition of the proposition: "when a polygon is a right pentagon, there is no parallel set of sides."

4.3 Expanding the content of spatial geometry curriculum

Applying the epistemological impact of 3DDGS it is possible to construct objects based on the properties and relationships of the figures used, and it even enables the construction of objects whose physical construction and manipulation are considered rather difficult. It is also possible to transform the constructed object while maintaining its attributes and then observing the transformation in multifaceted ways. And as a result of this, we can open up the potential that the contents whose learning has been conventionally difficult can be included in spatial geometry curriculum.

For example, as described above, the Japanese government educational curriculum guideline (revised in 1998) includes solids of revolution at seventh grade level. In this level students are intended to re-learn space figures (such as cones and cylinders), which had already been learned at elementary schools, as the figures constructed through rotation. Hence the guideline does not actually include constructing any new space figures. 3DDGS makes it possible to set the axis of revolution either inside or outside a plane figure and then to tilt that axis freely because there are no physical restraints or limits. This then enables the construction of a torus by setting the axis of revolution outside the circle, for example (Figure 3).

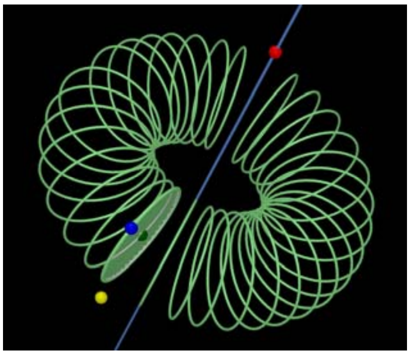
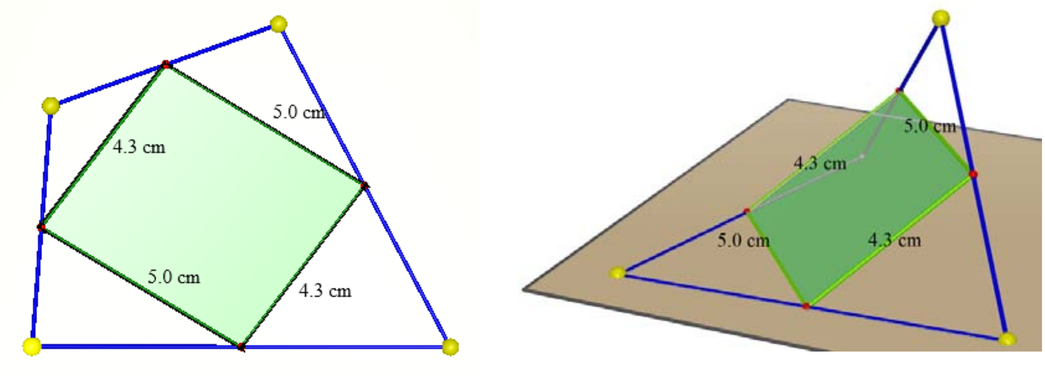


Fig. 3 Torus construction

Moreover, applying the epistemological impact of 3DDGS deepens relational understanding. Hence it is possible to generalize already-leaned figures' properties and relationships from the two-dimensional world and establish them three-dimensionally. And conversely it is possible to specify properties and relations in the three-dimensional world and then establish them two-dimensionally, which then enables elaborating learning contents beyond dimensions.

For example, with the Midpoint Theorem, we can prove that a quadrilateral created by connecting the midpoints of each side becomes a parallelogram (Figure 4, left). This proof does not use the fact that the four apexes of the quadrilateral are on the same plane. It makes it possible to predict that, even when four apexes are not on the same plane, a quadrilateral created by similarly sequentially connecting the midpoints will be a parallelogram. In fact, using 3DDGS, when one of the four apexes is redefined as a point three-dimensionally and then moved somewhere not on the same plane as the other three apexes, it is possible to confirm that the quadrilateral created by sequentially connecting the midpoints remains a parallelogram due to the fact that the two sets of the opposite sides are of equal length.



Two dimensional case Three dimensional case

Fig. 4 Quadrilaterals created by connecting midpoints

4.4 Strengthening the connection of content in different units

Applying the epistemological impact of 3DDGS enables connecting content of a unit with content of other units. This results in the integration of contents among different areas of school mathematics and in the advanced learning.

For example, the Japanese government educational curriculum guideline (revised in 1998) includes “circles” in the content of geometry at fourth grade level. At seventh grade level the content includes inverse propositions and their graphs and deals with the term, “hyperbolas.” Moreover, at eighth grade level the content includes quadratic functions and their graphs and deals with the term, “parabolas.” However, circles, hyperbolas and parabolas are not integrated as intersections constructed by cutting the surfaces of cones.

In contrast, 3DDGS enables the construction of cut surfaces in shapes that include circles, hyperbolas, parabolas and ellipses by working out ways to cut cones with a plane. In particular, hyperbolas are formed when cones are cut with a plane parallel to the axis of revolution. If the space is dragged in 3DDGS so as to make the generating line the coordinate axis, it is possible to observe a hyperbola on the rectangular coordinate. It also clarifies that the generating line of cone is an asymptote (Figure 5). This learning will support conducting advanced exploration to construct parabola-shaped cut surfaces when the term, "parabola," can be used to name a quadratic function graph in the eighth-grade.

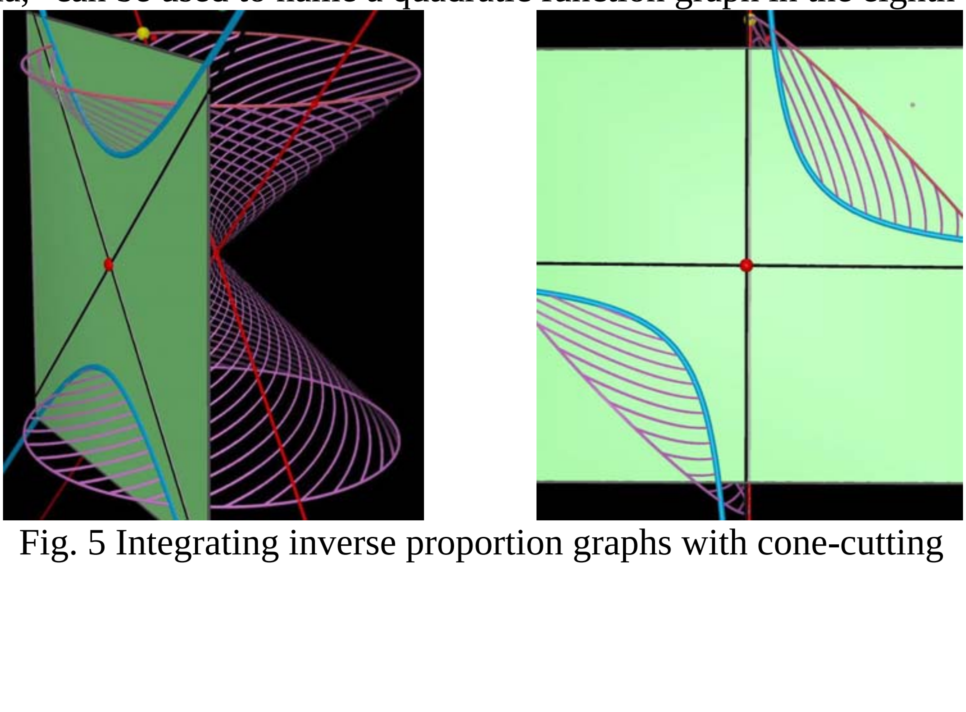


Fig. 5 Integrating inverse proportion graphs with cone-cutting

4.5 Strengthening the relationships between mathematics and real world

Applying the epistemological impact of 3DDGS enables mathematically modeling things around us and their behavior. This modeling process can clarify mathematical structure and mechanism embedded in things and their behavior, and can strengthen the correspondence between real world and mathematics.

For example, as a general rule butterflies symmetrically flap their wings using their bodies as the axis. 3DDGS models this movement as follows. First, straight line *l* is constructed on plane *p*, and circle *O* vertical to the plane is constructed centering around a point on the straight line. Next, a small arc is constructed on the circumference. This arc is the range within which the wings move. Next, point *A* is constructed on the arc, and the shape of a butterfly wing constructed with a polygon on plane *q* on which point *A* and straight line *l* are located. Finally, surface *r* vertical to plane *p* and passing through straight line *l* is constructed, and the other wing constructed by moving the wing (polygon) from plane *q* to plane *r* in a planar-symmetrical fashion. In fact, when point *A* on the arc is moved with the animation function, a butterfly starts flapping its wings (Figure 6). However, when they are flapping both wings sometimes overlap each other. This happens because the arc on circle *O* intersects symmetrical plane *r*. Hence upon correcting this both wings flap like a butterfly.

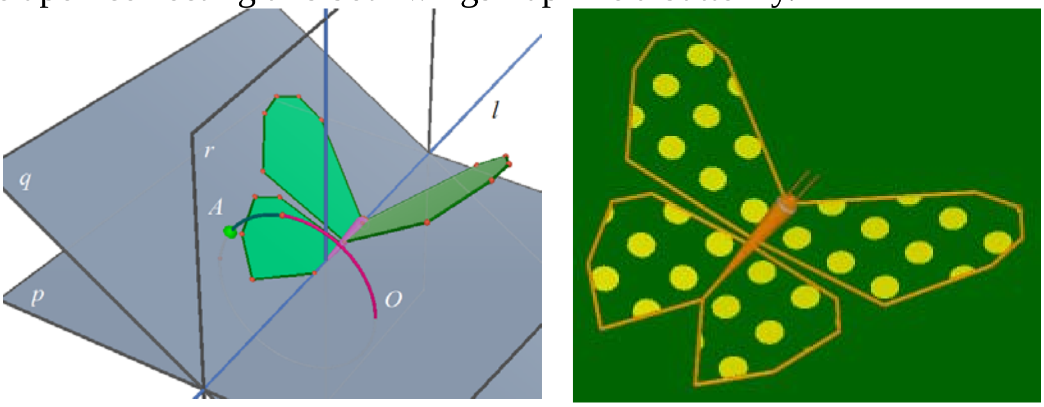


Fig. 6 Mathematical mechanism of a wing flapping

5. CONCLUDING REMARKS

Conclusion of this research is as follows.

By applying the epistemological impact of 3DDGS, the following potentials can be opened up in the curriculum development of spatial geometry in lower school mathematics.

- Empowering the learning activities in spatial geometry
- Empowering activities wherein space figures are logically explored
- Expanding the learning content of spatial geometry
- Strengthening the connection of content in different units
- Strengthening the relationships between mathematics and real world

Based on the above-mentioned potentials, a pilot spatial geometry curriculum for the seventh grade (14 hours in total) was developed and implemented. The achievement situation of students was then examined by using the problems and questionnaire of National Survey of Implementation and Achievement of Japanese government educational curriculum guideline. The results showed remarkable improvements in their understanding of space figures and their explanation about the geometrical properties in space (Chino et al., 2007).

The following are issues for further research:

- Is it possible to concretize the potentials of the epistemological impact of 3DDGS with spatial geometry curriculum development?
- Is the intended curriculum sufficiently meaningful for implementation when compared to the conventional curriculum?
- How will students' understanding of space figures change with the implemented curriculum?

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